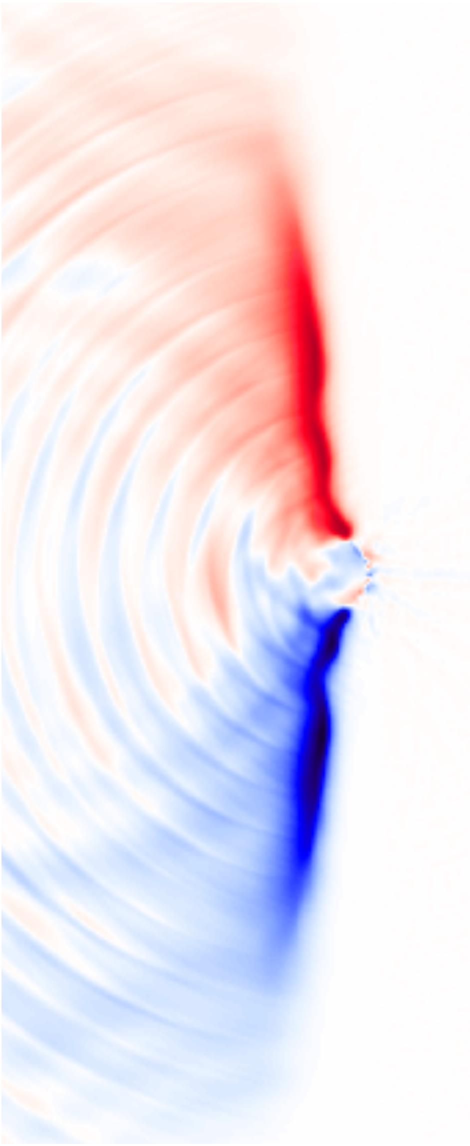
Kinetic solution for the generation of magnetic fields via the Biermann battery



Kevin Schoeffler[1]

Nuno Loureiro[2], Luis Silva[1], Ricardo Fonseca[1,3]

[1] GoLP / Instituto de Plasmas e Fusão Nuclear
Instituto Superior Técnico, Lisbon, Portugal

[2] Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge MA 02139, USA

[3] Instituto Universitário de Lisboa (ISCTE-IUL), Lisbon, Portugal

epp.tecnico.ulisboa.pt || **golp**.tecnico.ulisboa.pt

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Magnetic fields ubiquitous in astrophysics

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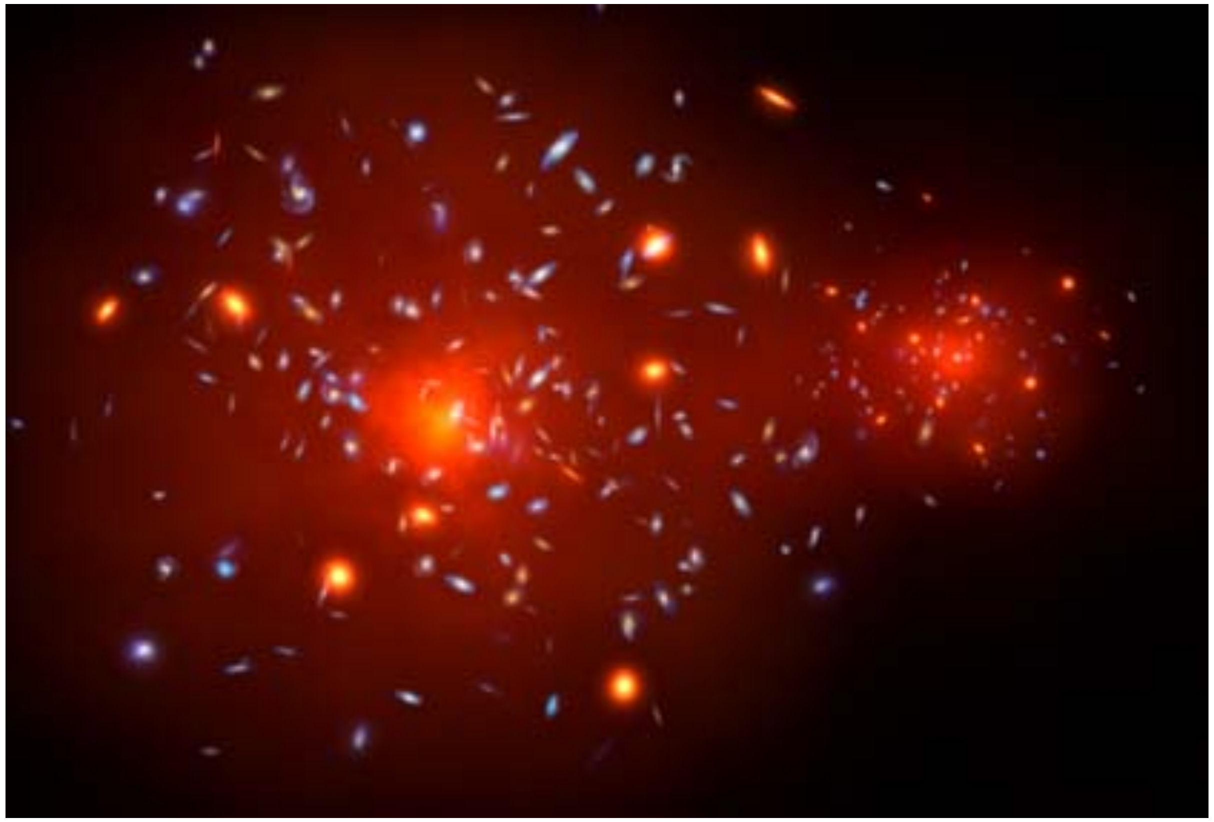
Crab Nebula

(NASA/HST/CXC/ASU/J. Hester et al.)

Cluster Merger



(Chandra x-ray observations)

Astrophysical magnetic field origin?

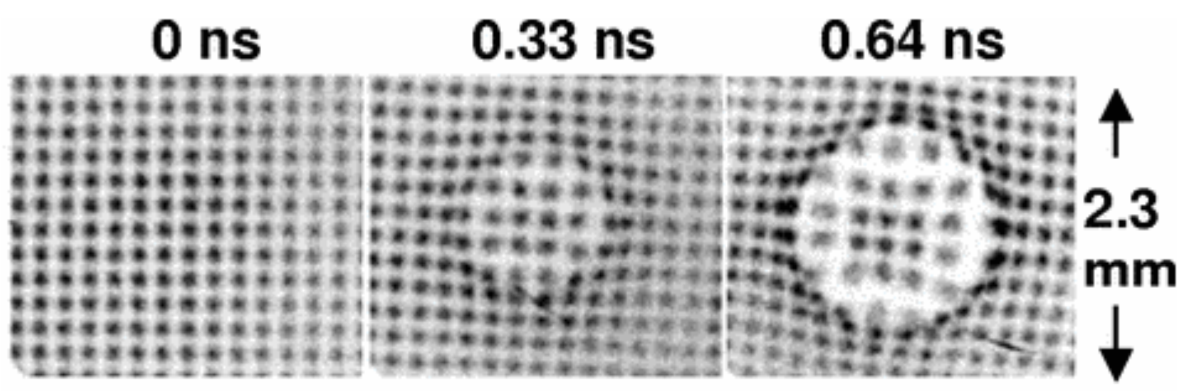
Biermann battery seed?

Collisionless environment

(Kulsrud 2008, 1992)

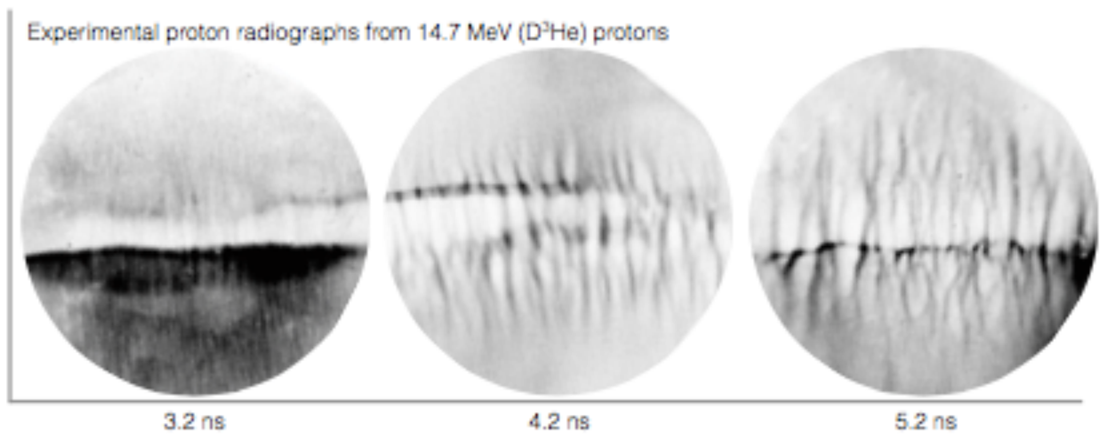
Mechanisms for B-fields in laser-plasmas

Biermann Battery



(Li et al. 2006)

Weibel instability



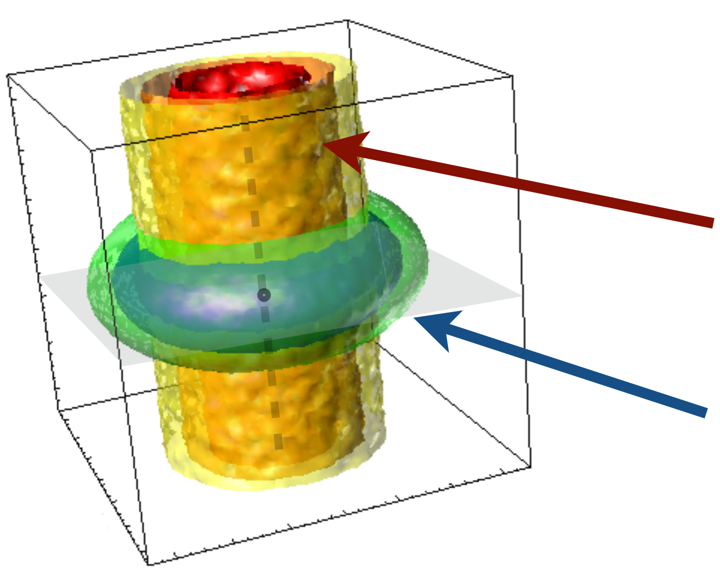
(Huntington et al. 2015)

What is the Biermann battery?

Initial state	Ingredients	Results
Plasma No magnetic fields	Density gradient: ∇n Temperature gradient: ∇T Perpendicular gradients: $\nabla n \times \nabla T \neq 0$	Magnetic field generated at gradient scale initially growing as $\frac{eB}{mc\omega_{pe}} = \lambda_D^2 \frac{\nabla n \times \nabla T}{nT} \omega_{pe} t$

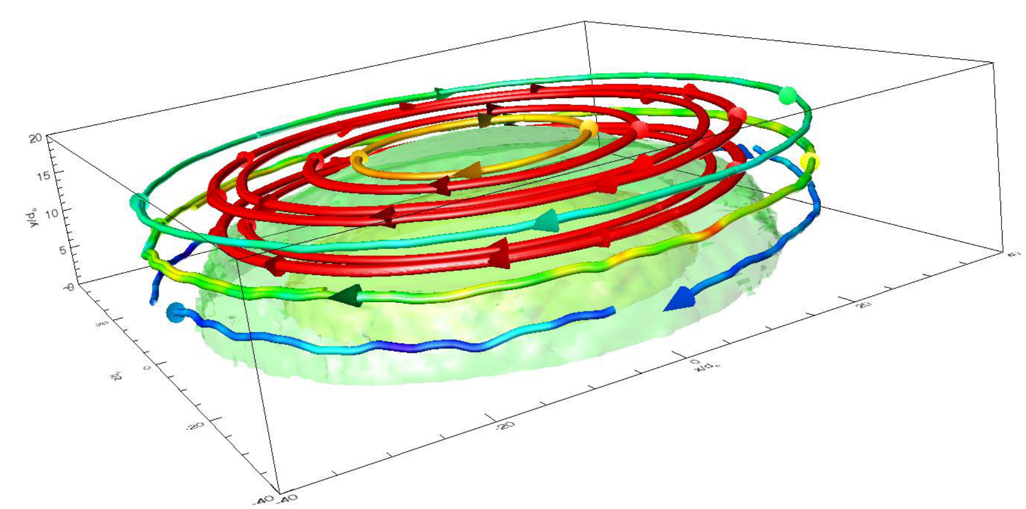
The Biermann Battery

Not an instability!



T contours

n contours



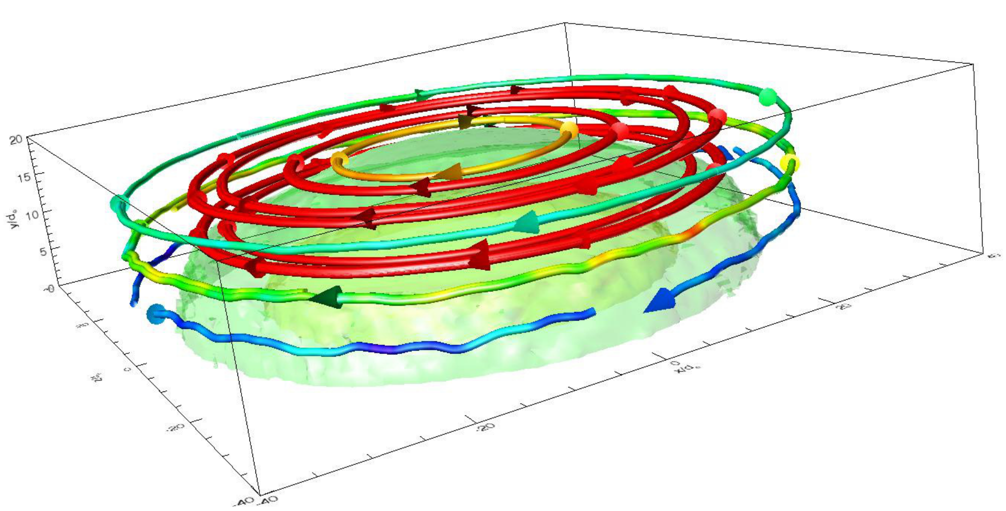
How strong are the expected fields

Biermann grows

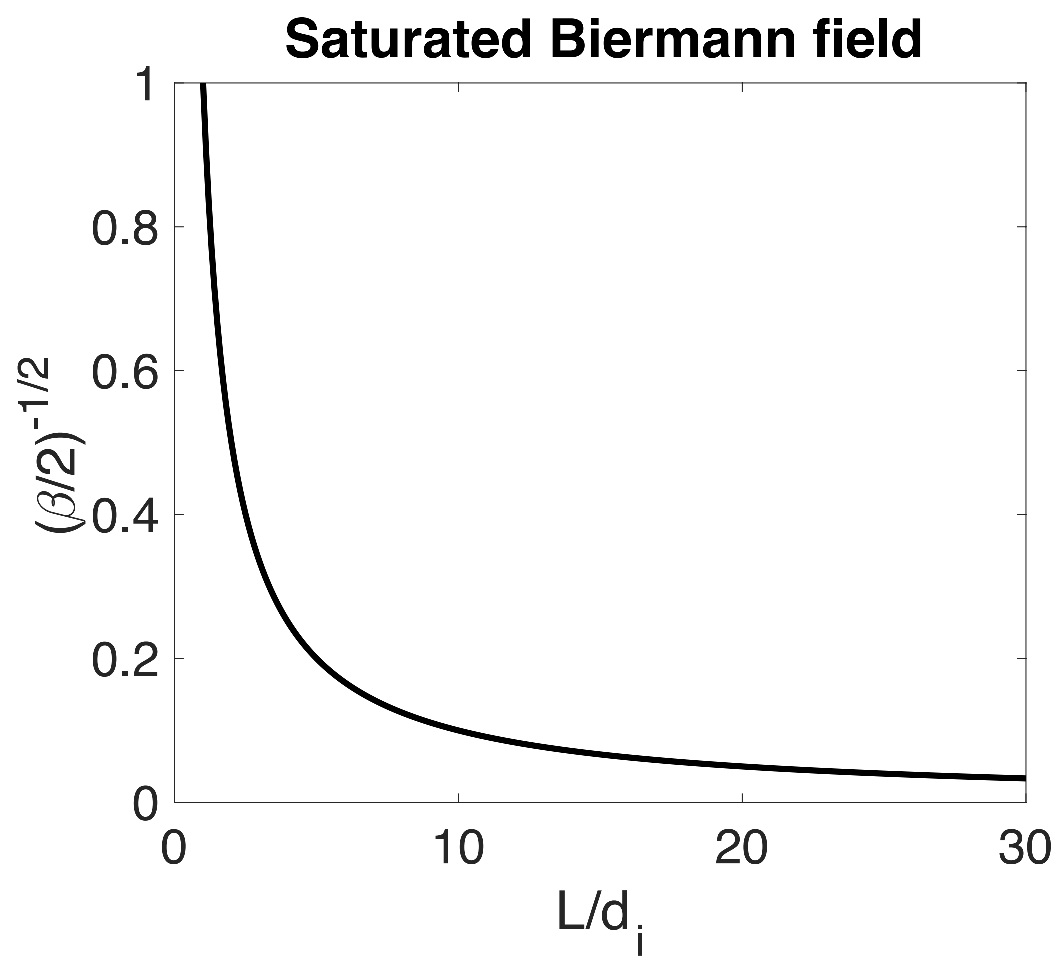
Magnetic field grows as

$$\frac{eB}{mc\omega_{pe}} = \lambda_D^2 \frac{\nabla n \times \nabla T}{nT} \omega_{pe} t$$

until …



Prediction of saturation



Normalized B = $\beta^{-1/2}$ predicted to follow 1/L scaling

(Haines 1997)

Osiris 3.0

osiris framework

- Massivelly Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium
  ⇒ UCLA + IST

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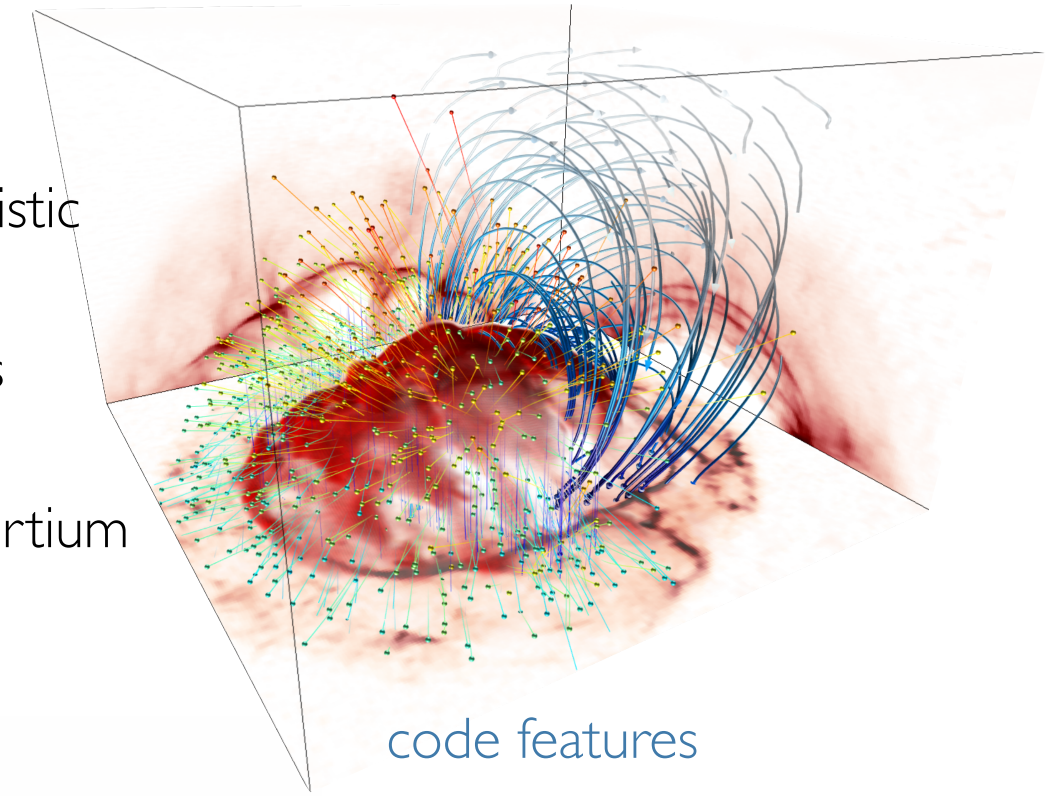
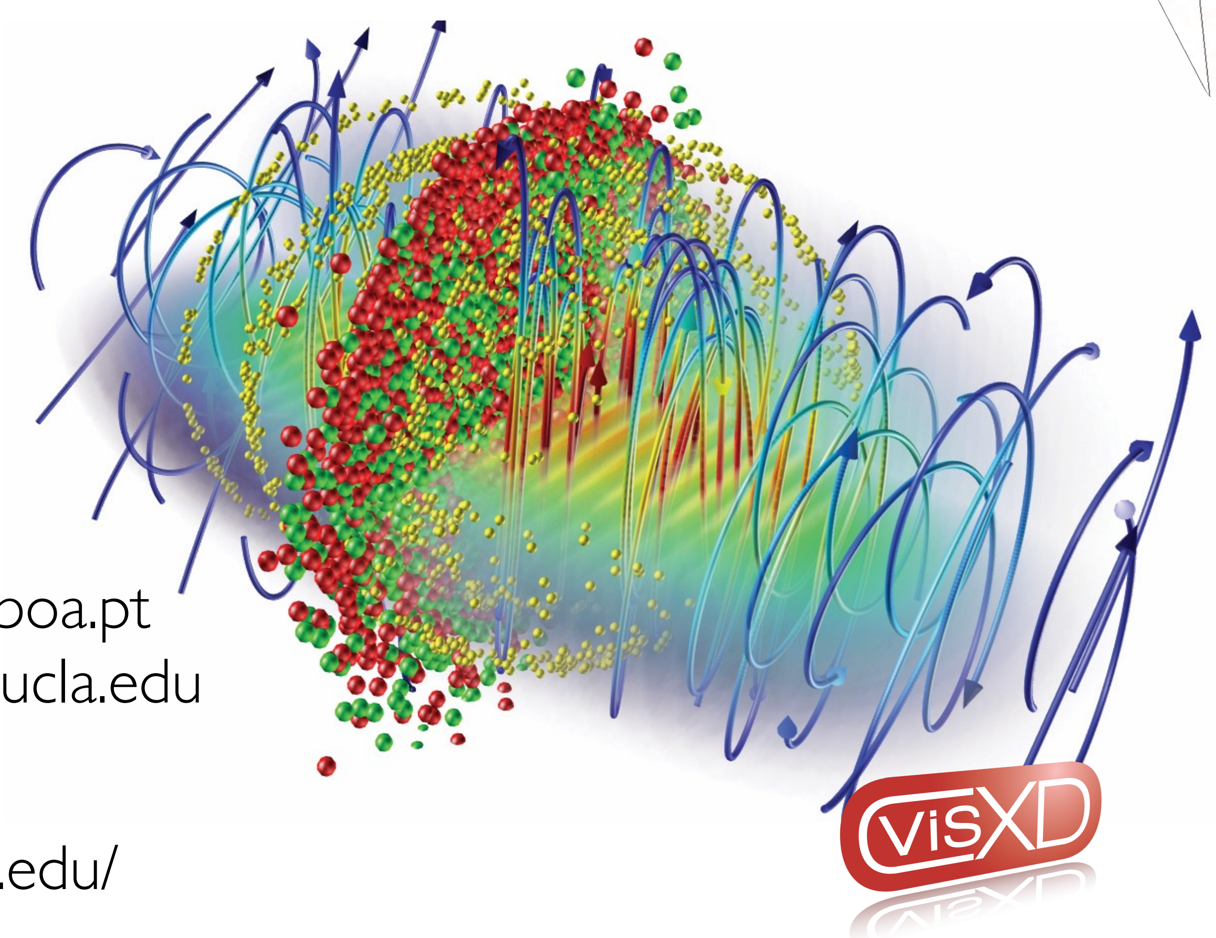




code features

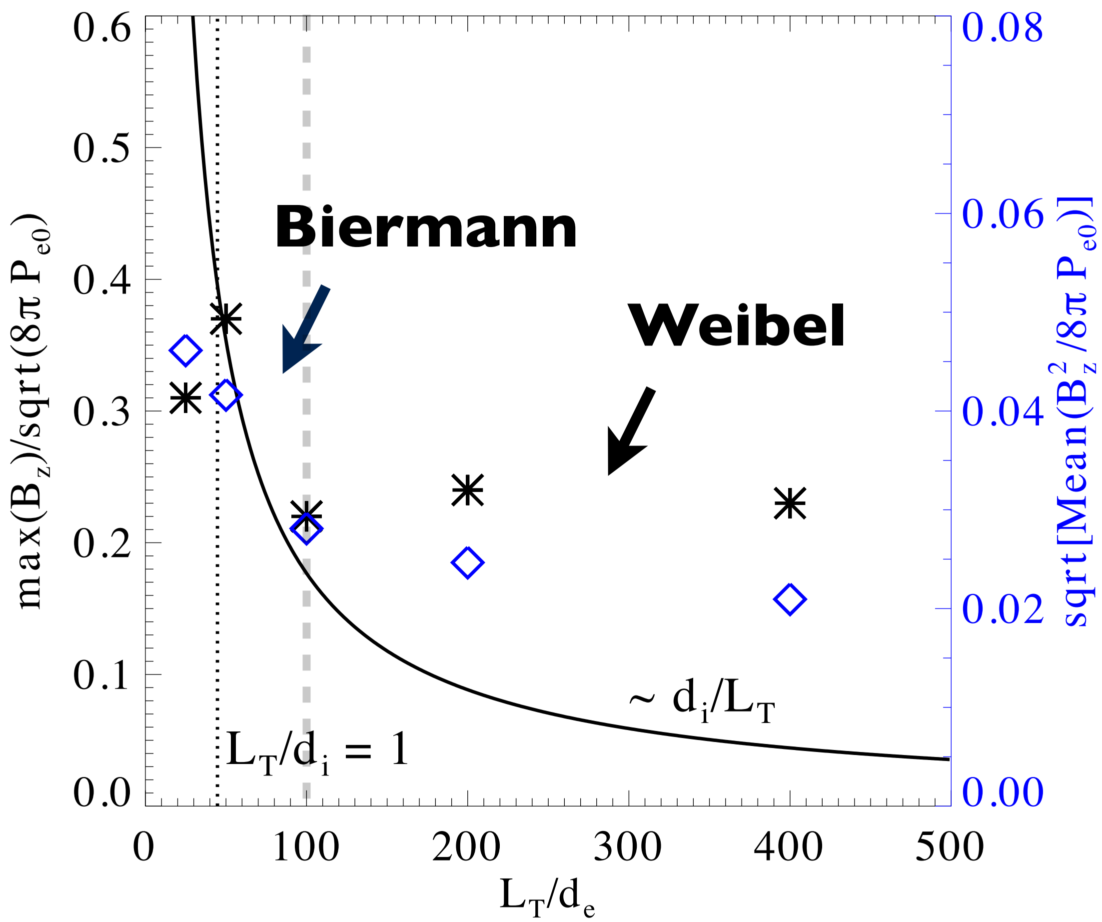
- Scalability to ~ 1.6 M cores
- SIMD hardware optimized
- Parallel I/O
- Dynamic Load Balancing
- QED module
- Particle merging
- GPGPU support
- Xeon Phi support

Ricardo Fonseca: ricardo.fonseca@tecnico.ulisboa.pt
Frank Tsung: tsung@physics.ucla.edu

http://epp.tecnico.ulisboa.pt/
http://plasmasim.physics.ucla.edu/

Kinetic effects in collisionless systems

Scaling with system size

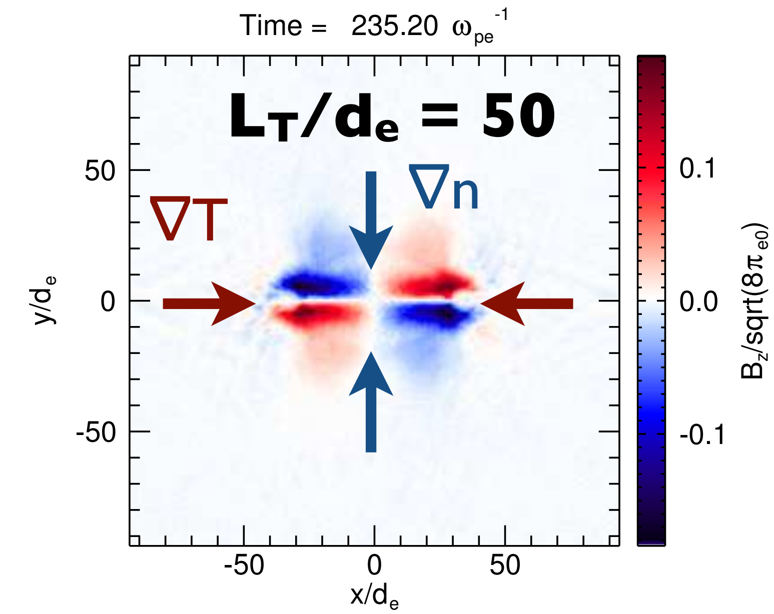


(Schoeffler et al. 2014, Schoeffler et al. 2016)

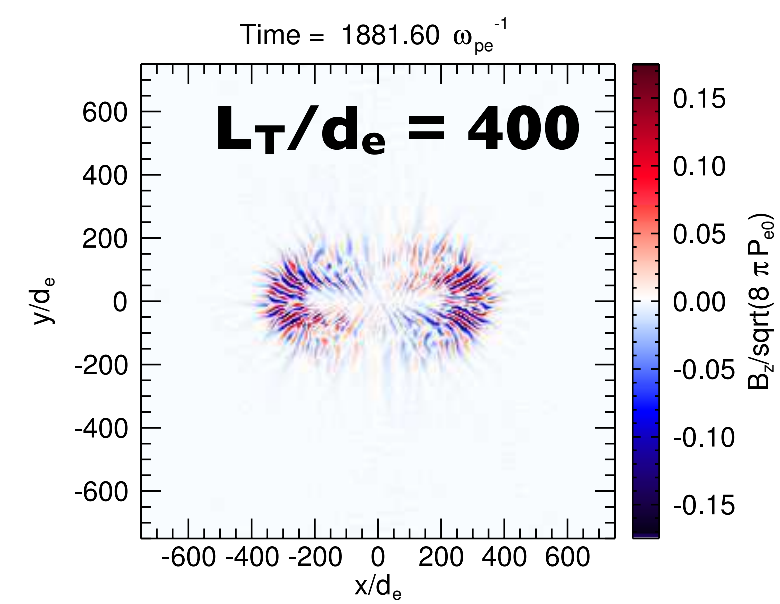
B follows 1/L scaling (Biermann regime) (Haines 1997)

then remains finite at large L (Weibel regime)

Biermann



Weibel



Can a kinetic solution be found?

Maxwell-Vlasov equations

$$\frac{\partial f}{\partial t} = -\mathbf{v}\cdot\nabla f + \frac{e}{m_e}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\nabla_v f$$

$$\frac{\partial \mathbf{E}}{\partial t} = c\nabla\times\mathbf{B} + 4\pi e\int dv^3\mathbf{v}f$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E}$$

Maxwell Distribution

$$f(t=0) = f_M(v_x, v_y, v_z)$$

Perturbed by gradients:

$$n = n_0\left(1 + \epsilon\frac{x}{\lambda_D}\right)$$

$$T = T_0\left(1 + \delta\frac{y}{\lambda_D}\right)$$

Small parameters

$$\epsilon = \frac{\lambda_D\nabla n}{n_0}$$

$$\delta = \frac{\lambda_D\nabla T}{T_0}$$

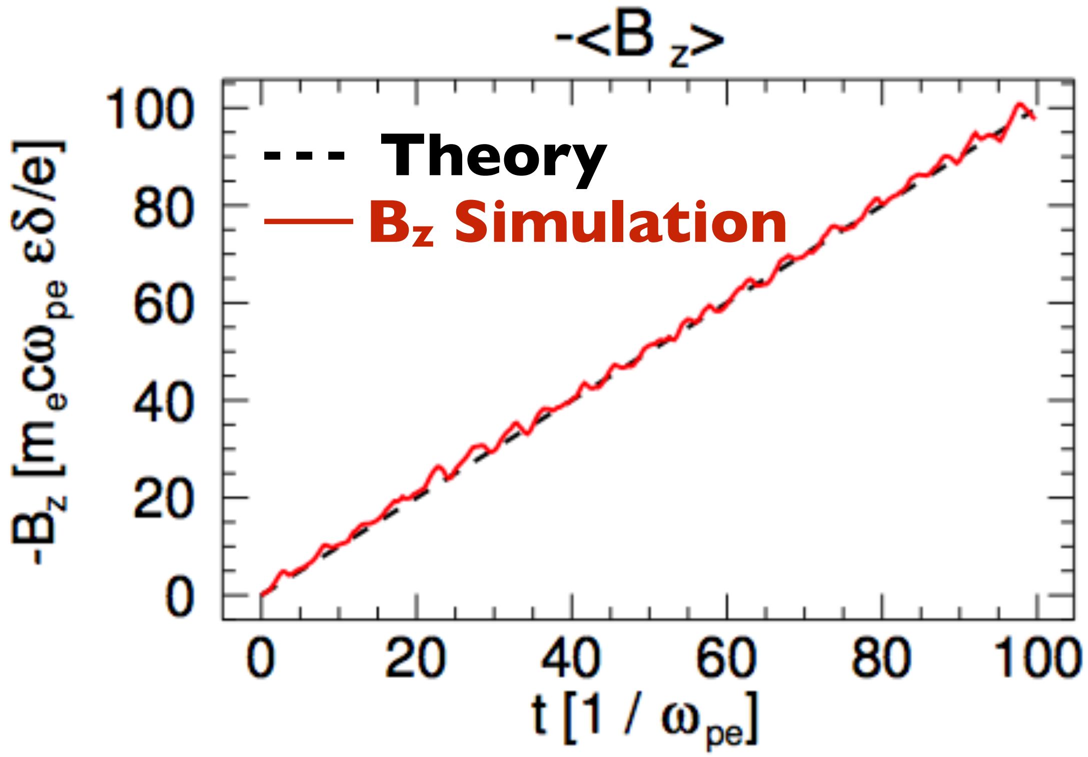
We found a kinetic solution!

(Schoeffler et al. 2017 arXiv 1707.06069)

Linear Biermann growth

$$\frac{eB_z}{mc\omega_{pe}} = -\epsilon\delta\omega_{pe}t$$

Equal to fluid predictions



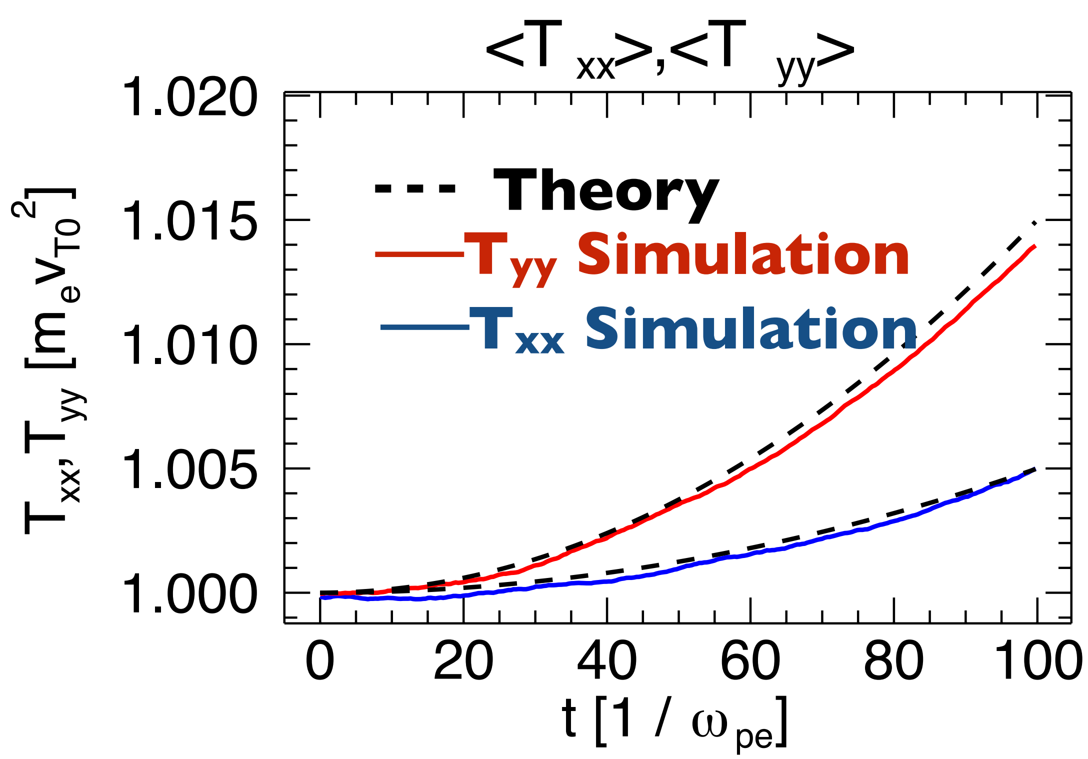
Temperature Anisotropy

$$A \equiv \frac{T_{yy}}{T_{xx}} - 1 = (\delta\omega_{pe}t)^2$$

Time scale $(\delta\omega_{pe})^{-1}$ is the crossing time L_T/v_{T0}

Strictly kinetic result

Leading to instabilities

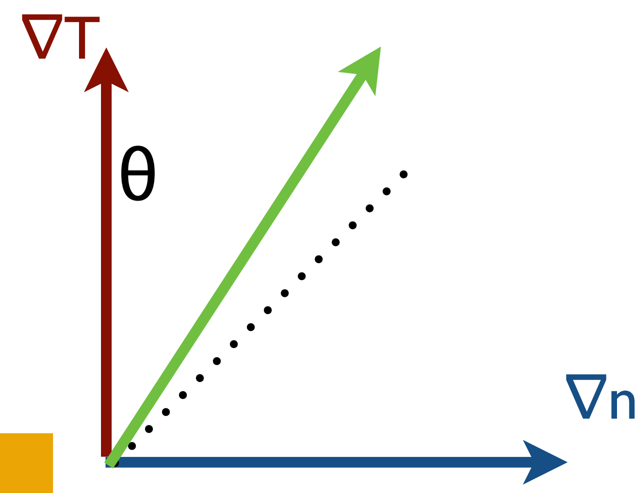


A temperature "tensor"

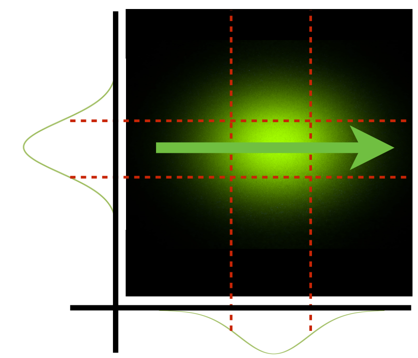
$$T_{ij} = \frac{m_e}{n}\int dv^3 v_i v_j f$$

Temperature Tensor

$$T_{ij} = T_0 + T_0 \begin{vmatrix} \delta^2 & \epsilon\delta \\ \epsilon\delta & 3\delta^2 \end{vmatrix} (t\omega_{pe})^2/2$$



Hot Direction



Rotated Temperature Tensor

$$T_{ij} = T_0 + T_0 \begin{vmatrix} 2\delta^2 - A_0 & 0 \\ 0 & 2\delta^2 + A_0 \end{vmatrix} (t\omega_{pe})^2/2$$

If you rotate clockwise by

$$\theta = \frac{1}{2}\tan^{-1}\frac{\epsilon}{\delta}$$

The matrix is diagonalized

Rotated Anisotropy

$$A = A_0 (t\omega_{pe})^2$$

$$A_0 = \delta(\delta^2 + \epsilon^2)^{1/2}$$

Most general perturbation

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$$n = n_0 \left(1 + \epsilon_\parallel \frac{x'}{\lambda_D} + \epsilon_\perp \frac{y'}{\lambda_D} + \epsilon^2 \kappa_{nij} \frac{x_i x_j}{\lambda_D} \right)$$

$$T = T_0 \left(1 + \delta \frac{x'}{\lambda_D} + \delta^2 \kappa_{Tij} \frac{x_i x_j}{\lambda_D} \right)$$

Arbitrary gradient angle

$\nabla n \cdot \nabla T \neq 0$

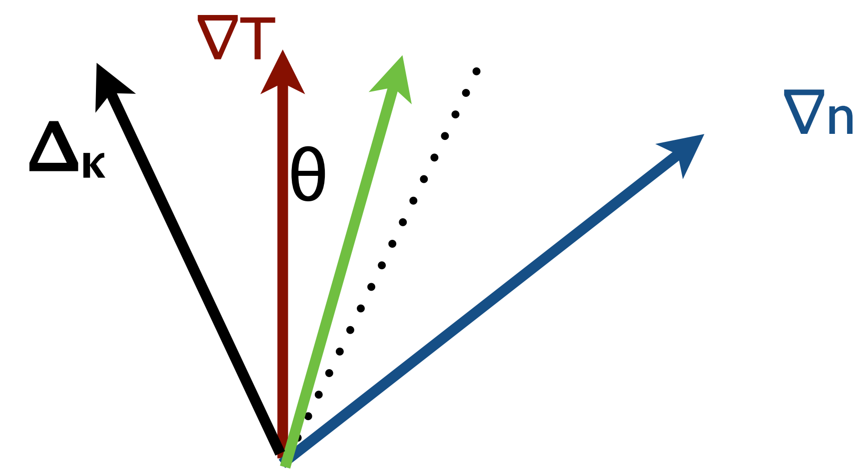
2nd order gradients

2nd order terms in δ and ε, proportional to κ_{Tij} and κ_{nij}

(Schoeffler et al. 2017 arXiv 1707.06390)

κ_{Tij} affecting the temperature anisotropy

$$T = T_0\left(1 + \delta\frac{x'}{\lambda_D} + \delta^2\kappa_{Tij}\frac{x_i x_j}{\lambda_D}\right)$$



2nd order T gradient tensor

$$\kappa_{Tij} = \begin{vmatrix} \kappa_{\parallel} & \kappa_{\times} \\ \kappa_{\times} & \kappa_{\perp} \end{vmatrix}$$

New vector affects anisotropy

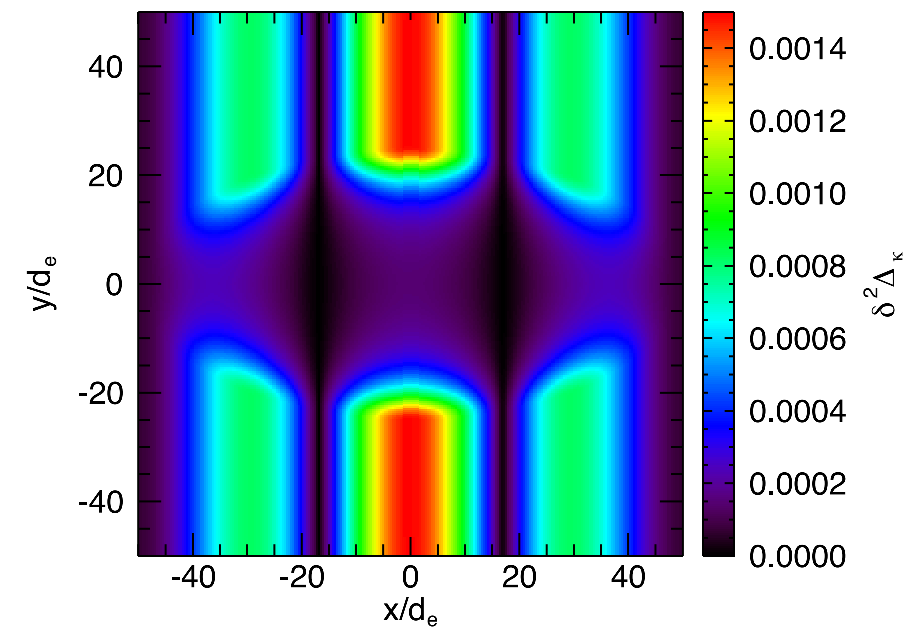
$$\Delta_\kappa = \begin{vmatrix} \Delta_{\kappa\parallel} \\ \Delta_{\kappa\perp} \end{vmatrix} = \begin{vmatrix} \kappa_{\parallel}-\kappa_{\perp} \\ 2\kappa_{\times} \end{vmatrix}$$

$$A = \delta|\delta + \varepsilon + \delta\Delta_\kappa|$$

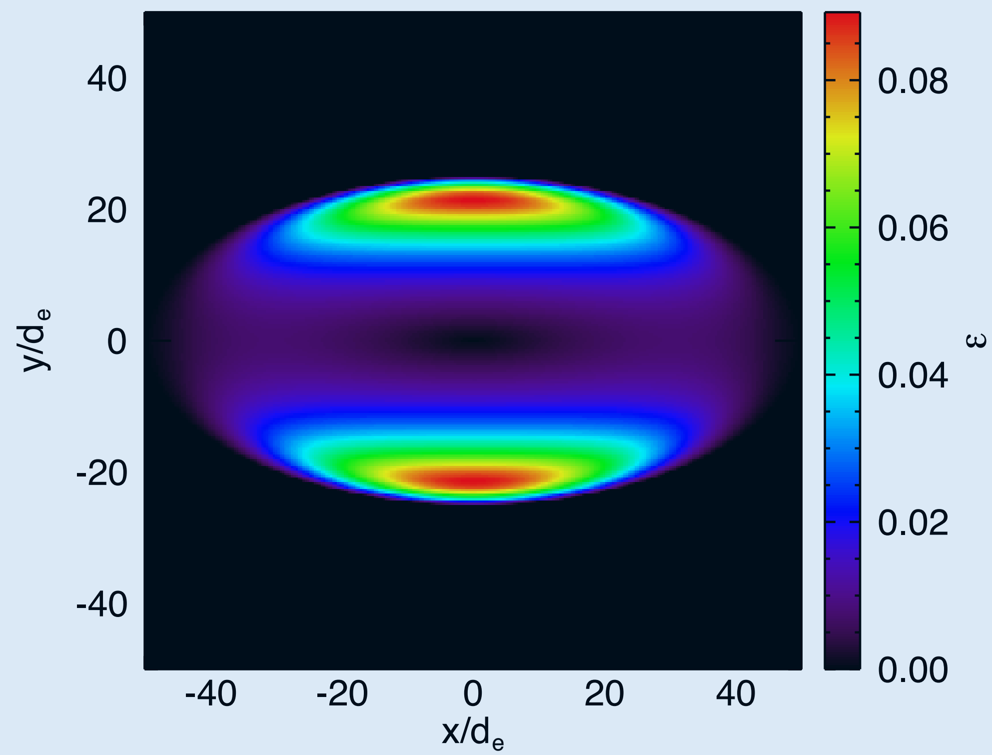
As a function of space

$$A = \delta|\delta + \varepsilon + \delta\Delta_\kappa|$$

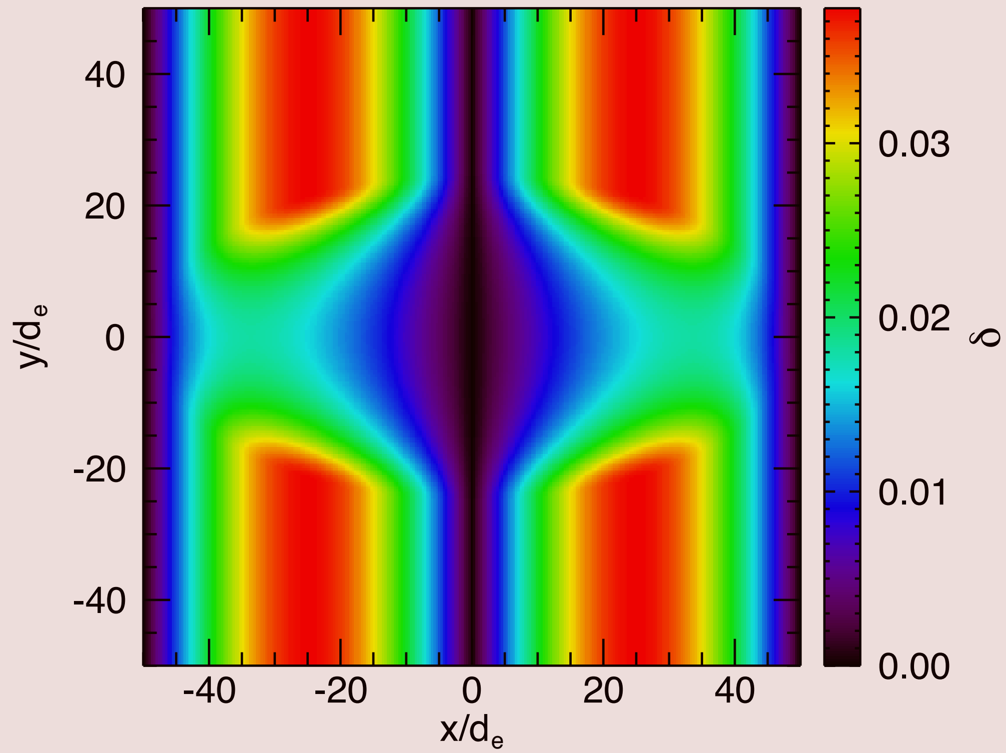
Spatial map of $\delta^2\Delta_\kappa$



Spatial map of ε



Spatial map of δ



We can predict anisotropy vs. space!

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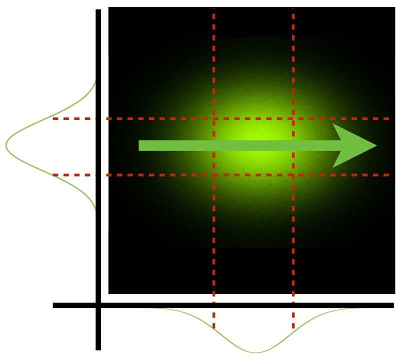
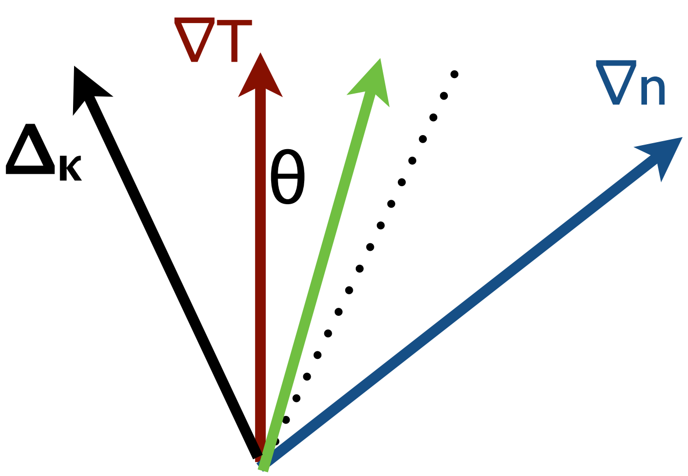
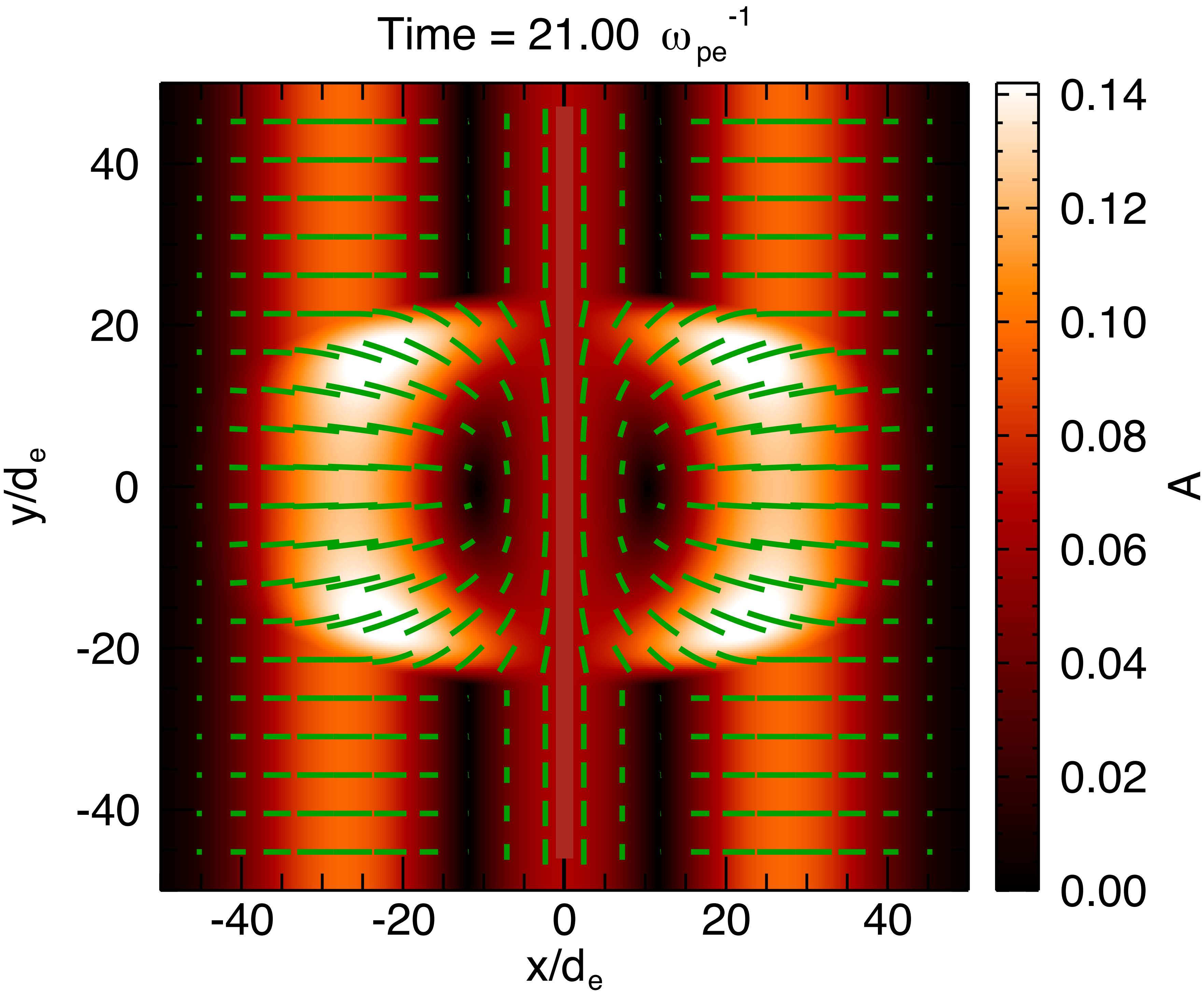
Hot Direction





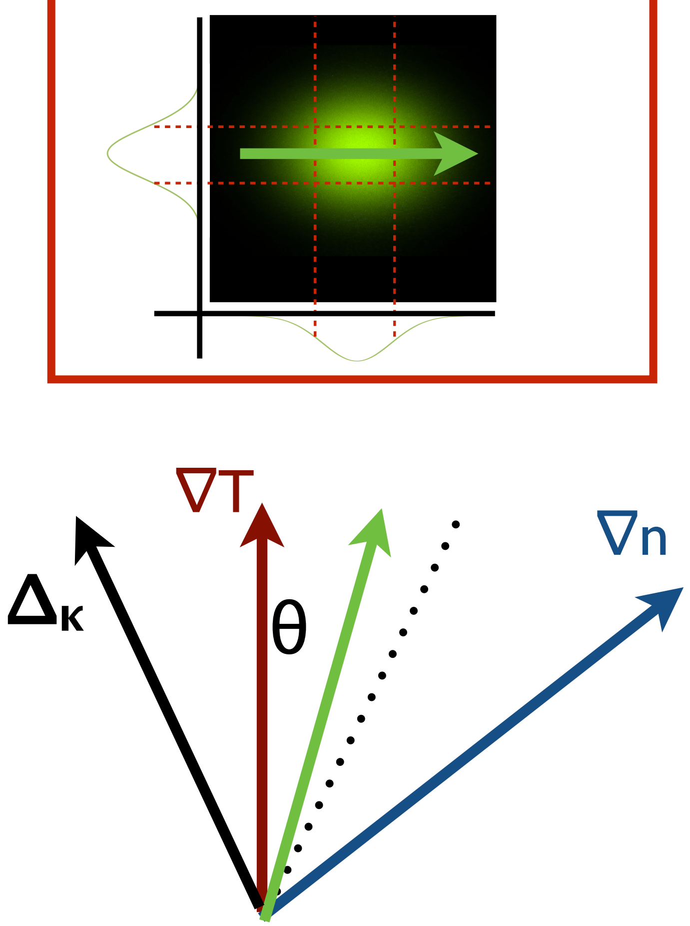
$\theta(x,y)$ and $A(x,y)$ can be found using $\varepsilon_\perp(x,y)$, $\varepsilon_\|(x,y)$, $\delta(x,y)$, $\kappa_{xx}(x,y)$, $\kappa_{xy}(x,y)$, and $\kappa_{yy}(x,y)$

Theoretical anisotropy

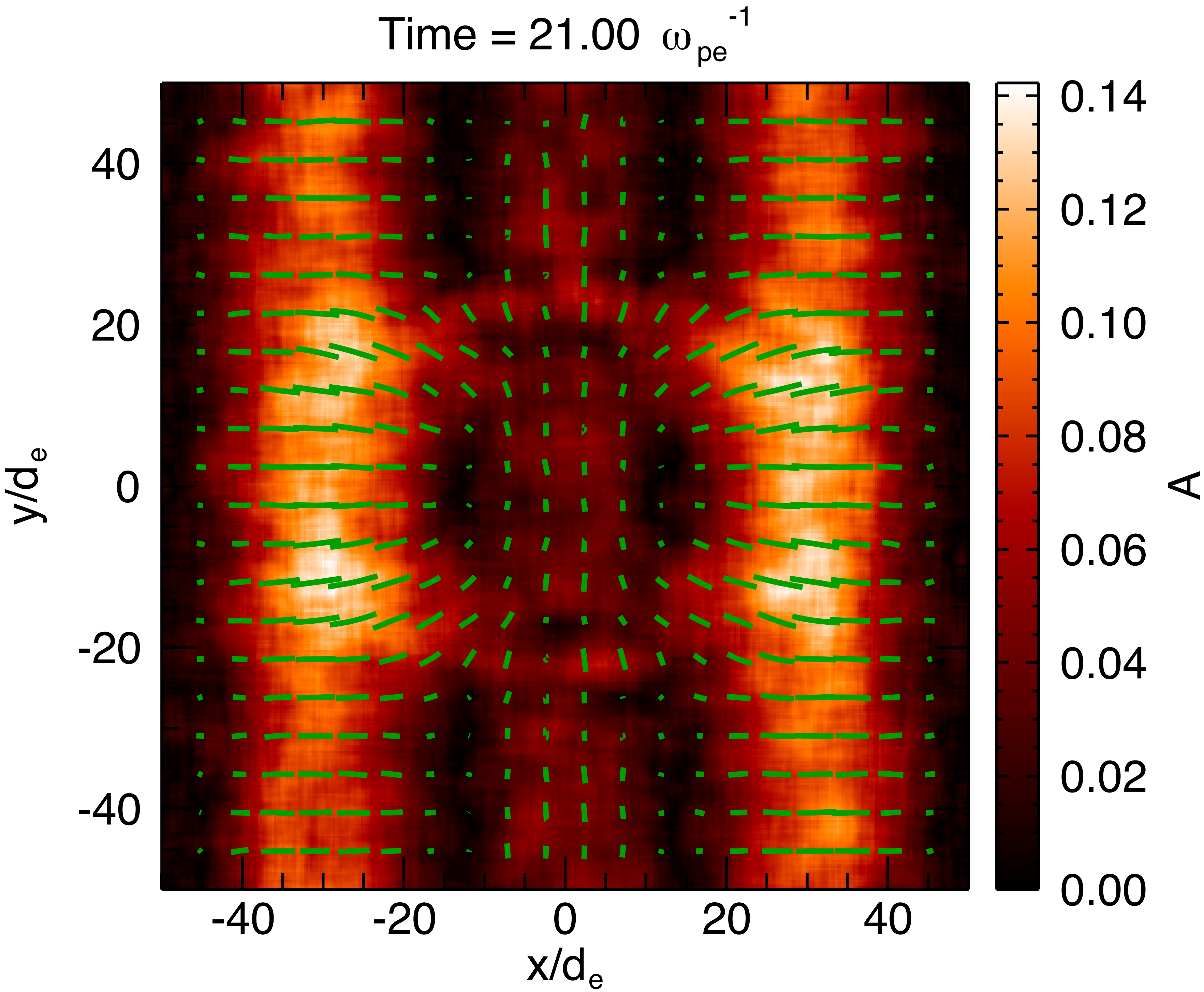


Does the anisotropy match simulations?

Hot Direction

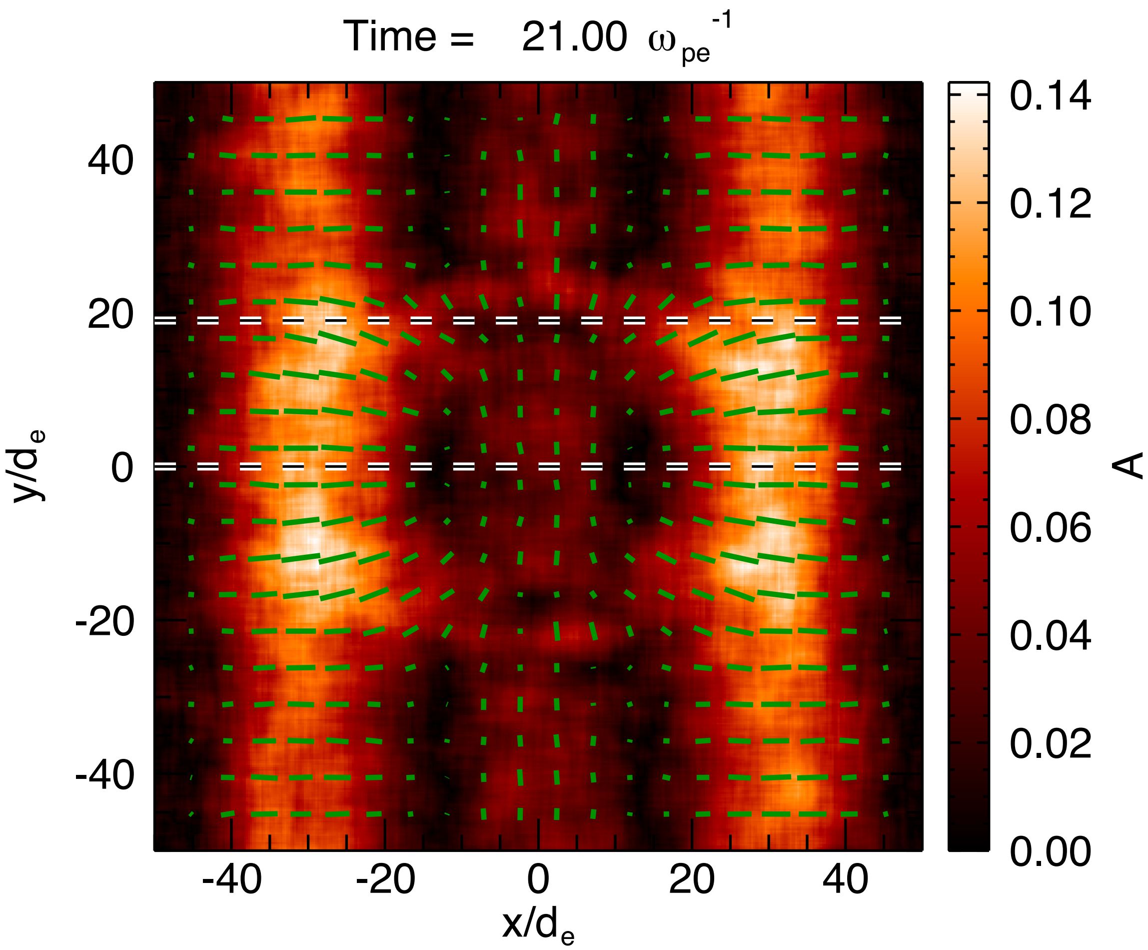


Simulated anisotropy



Does the anisotropy match simulations?

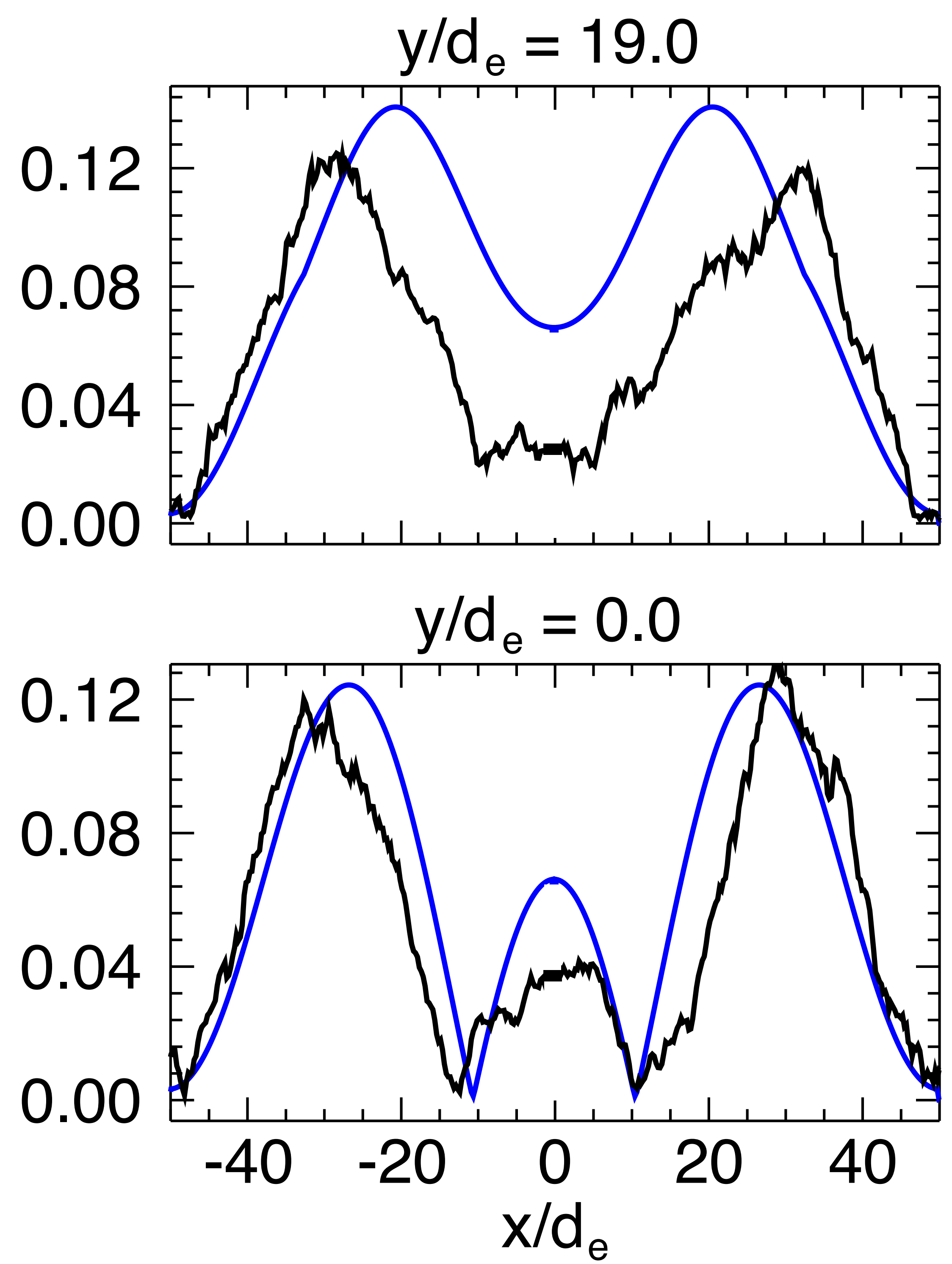
Looking closer



Simulation

Theory

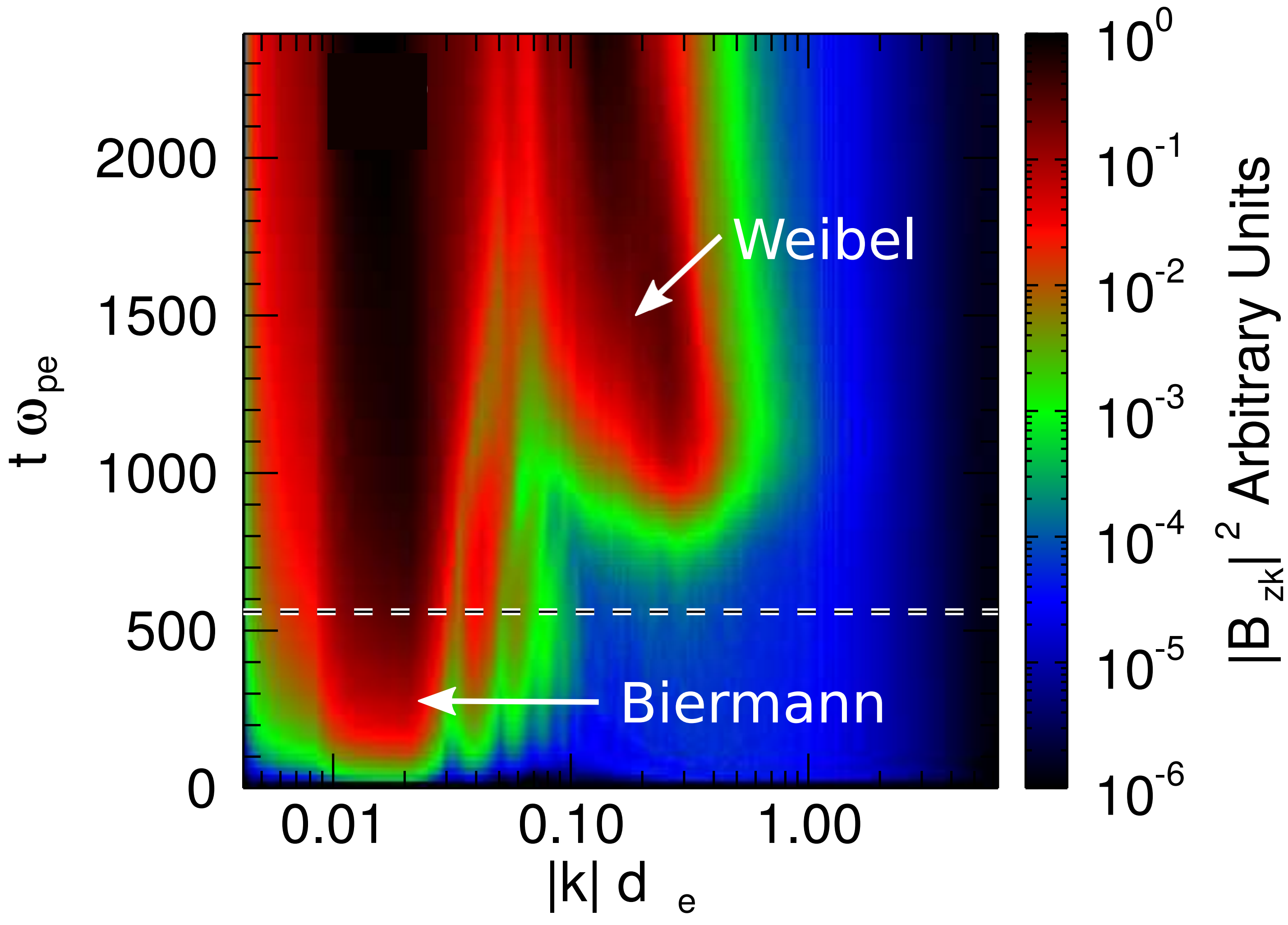
Theoretical/Simulated Anisotropy



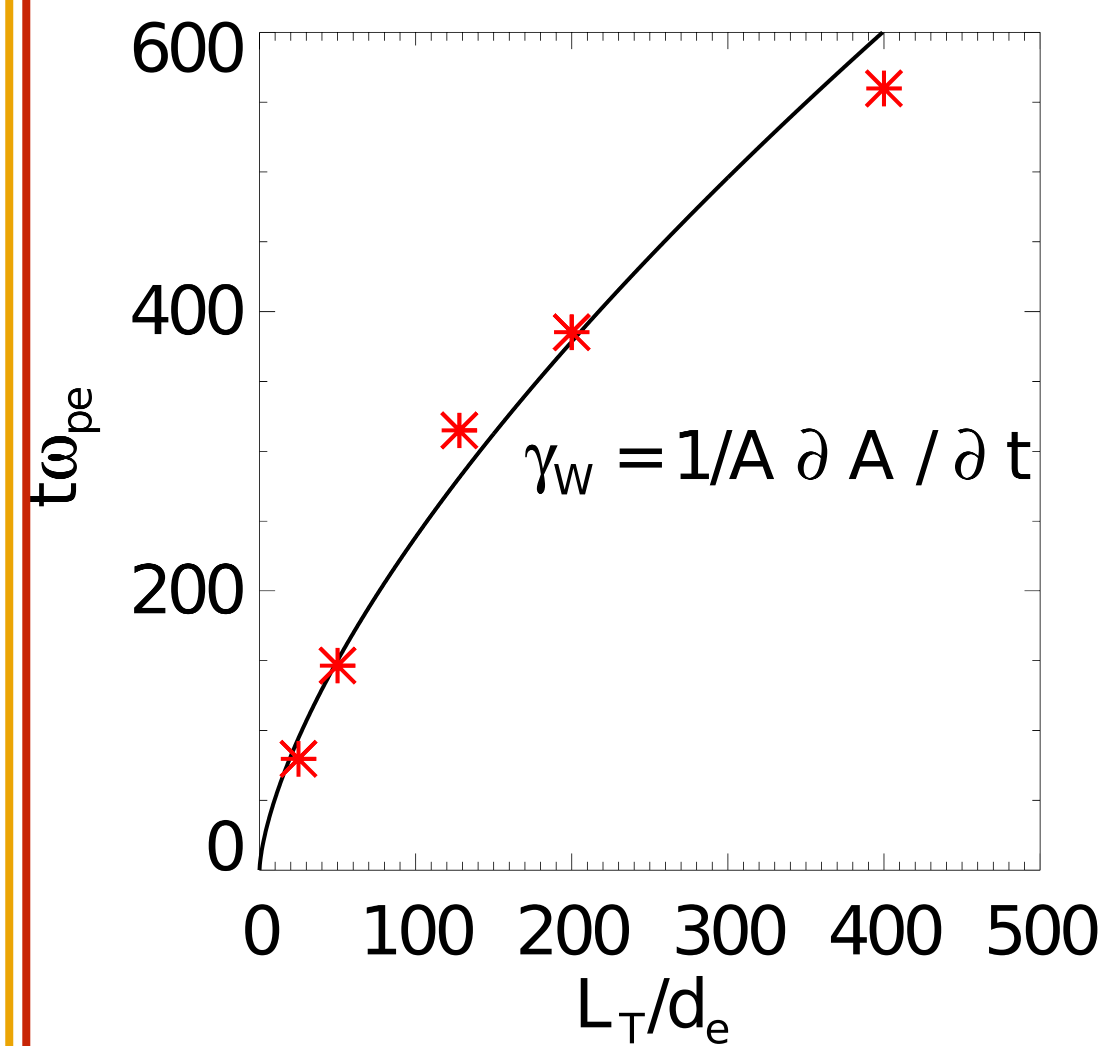
(Schoeffler et al. 2017 arXiv 1707.06390)

Weibel onset matches anisotropy growth

When does Weibel onset?

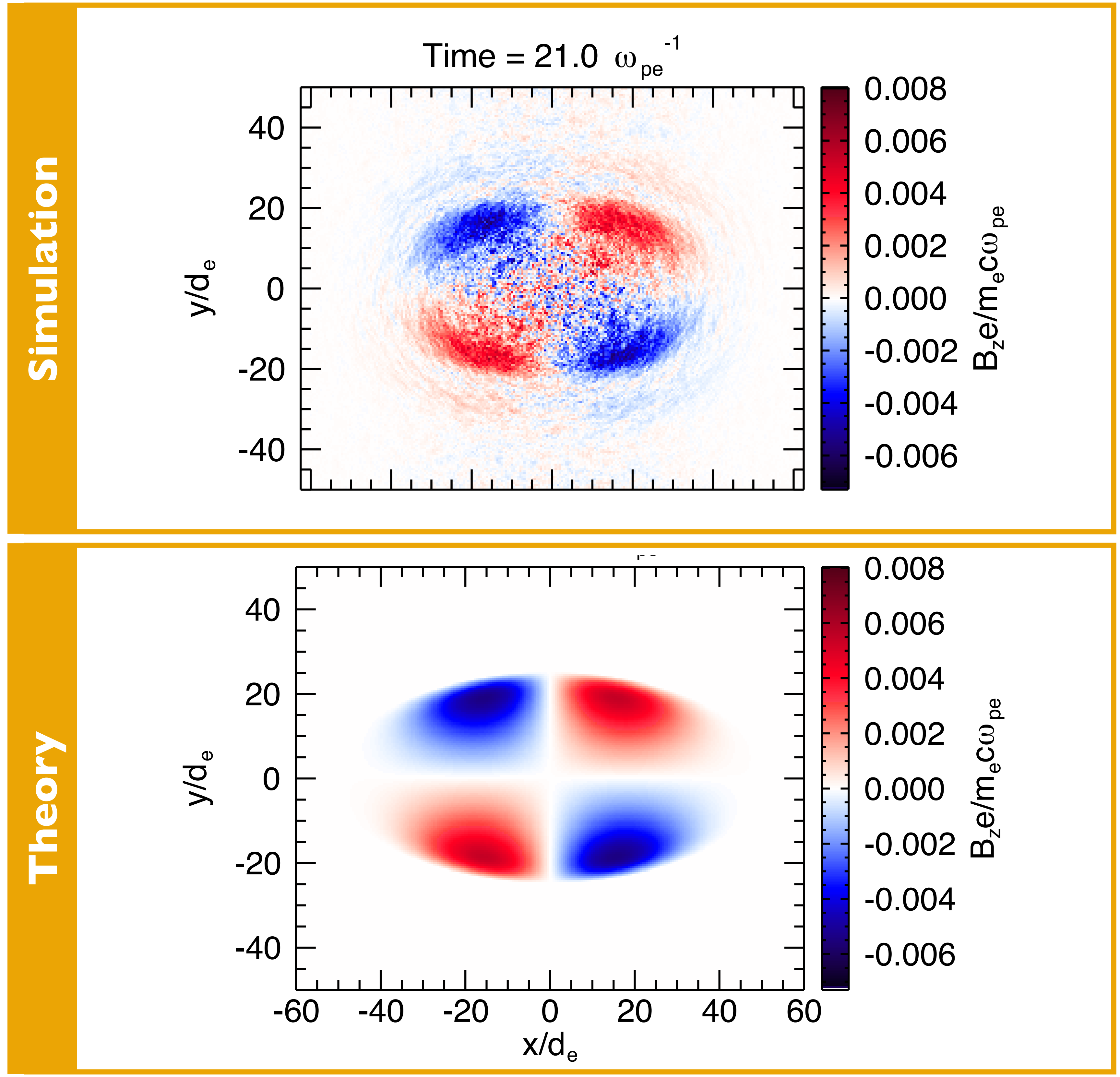


When B grows faster than A



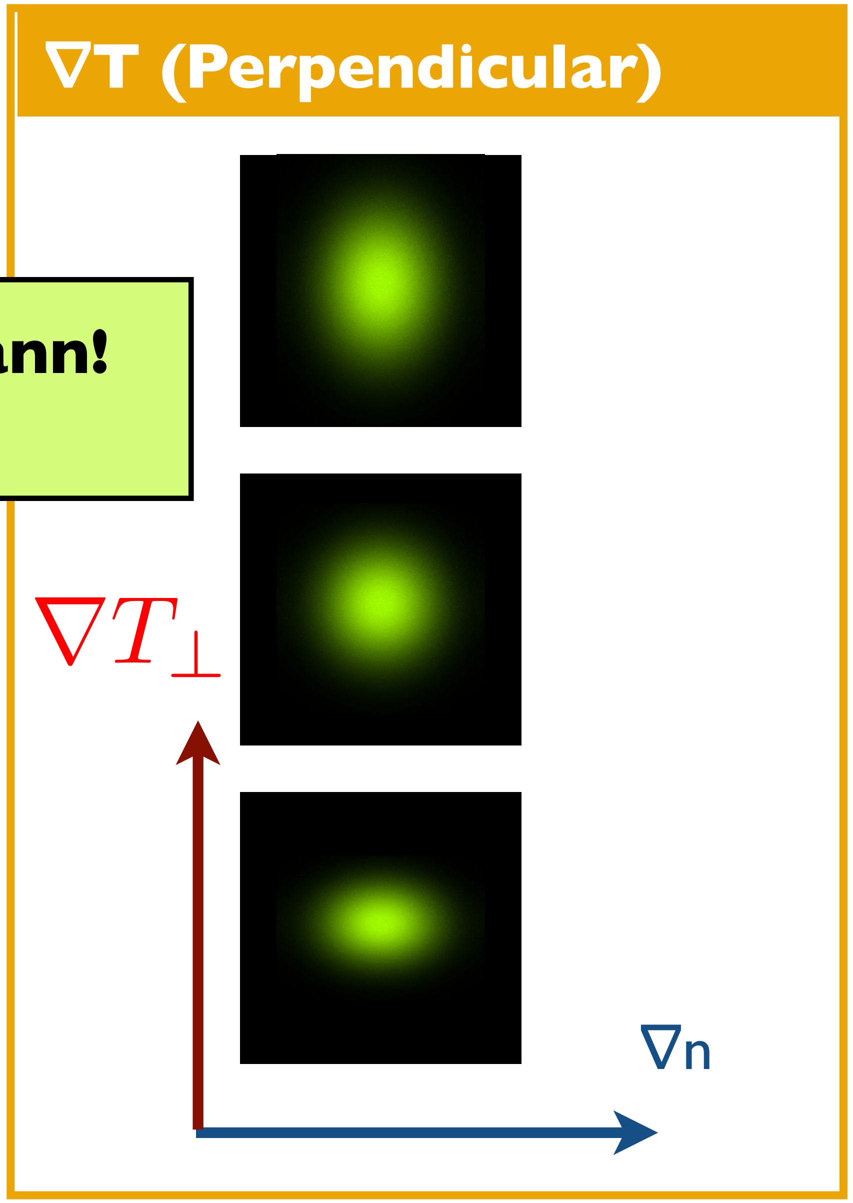
We can predict Biermann battery vs. space!

$$\frac{eB}{m_e c \omega_{pe}} = \epsilon_{\perp} \delta \omega_{pe} t$$

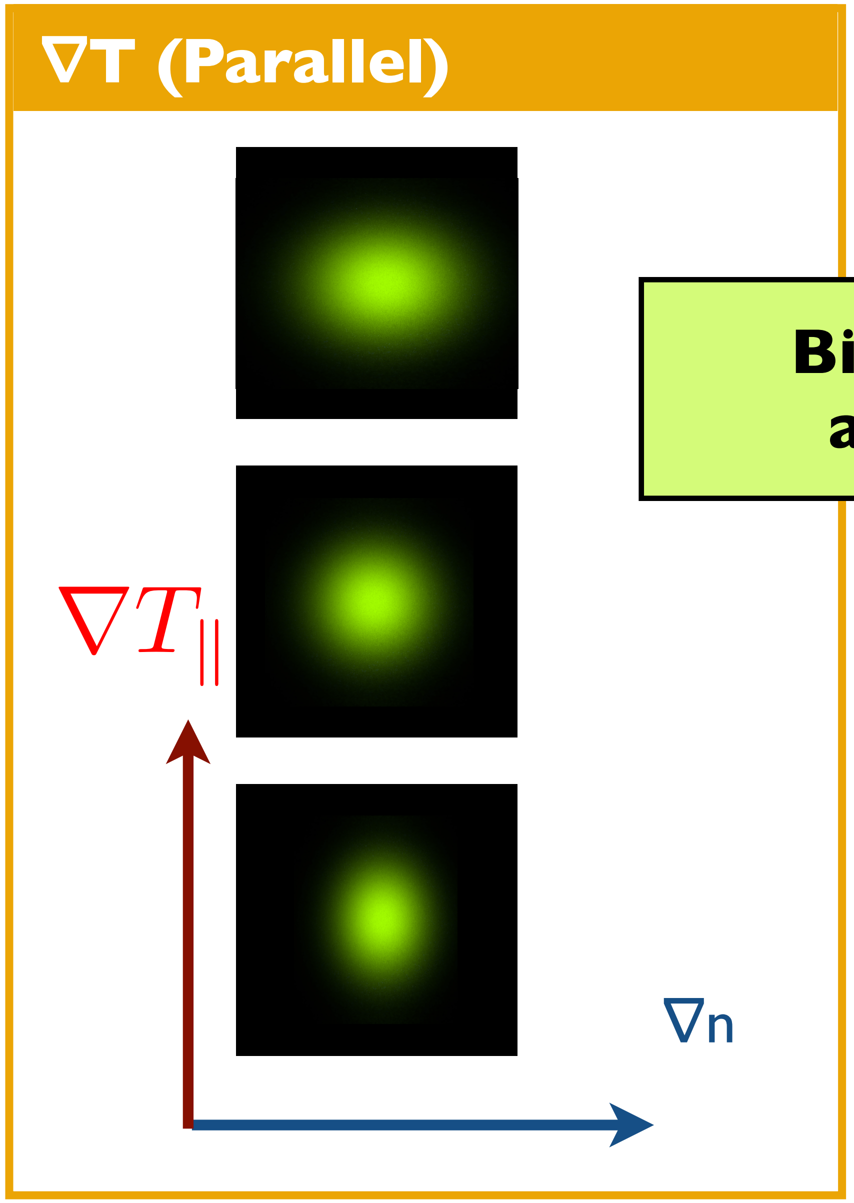


Predicted kinetic Biermann field

$$B = -\frac{m_e c^3}{e}\frac{\nabla n \times \nabla T_{\parallel}}{m_e n c^2} t$$



No Biermann!
XXX



Biermann allowed

Even relevant in magnetized space systems

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In essentially 1D flux tubes where

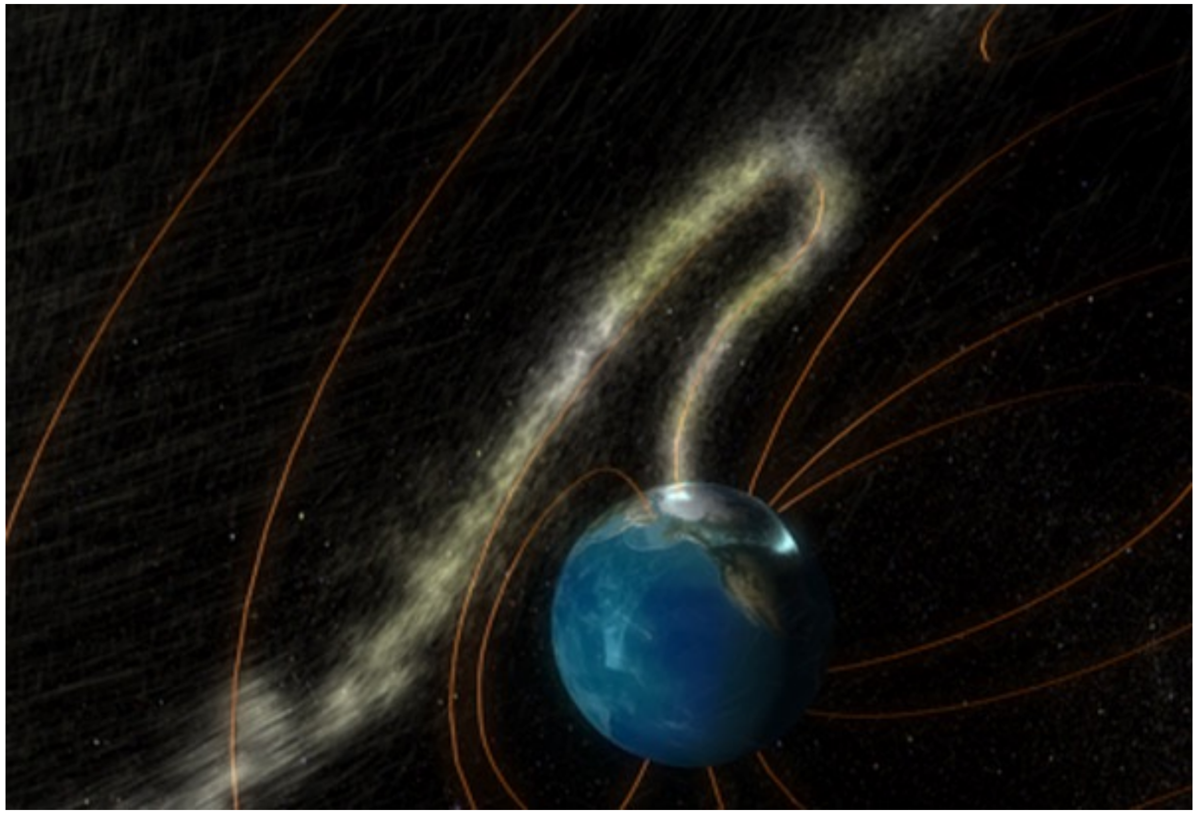
$\nabla T \parallel \mathbf{B_0} \parallel \nabla n \parallel \Delta_\kappa$

We predict an anisotropy

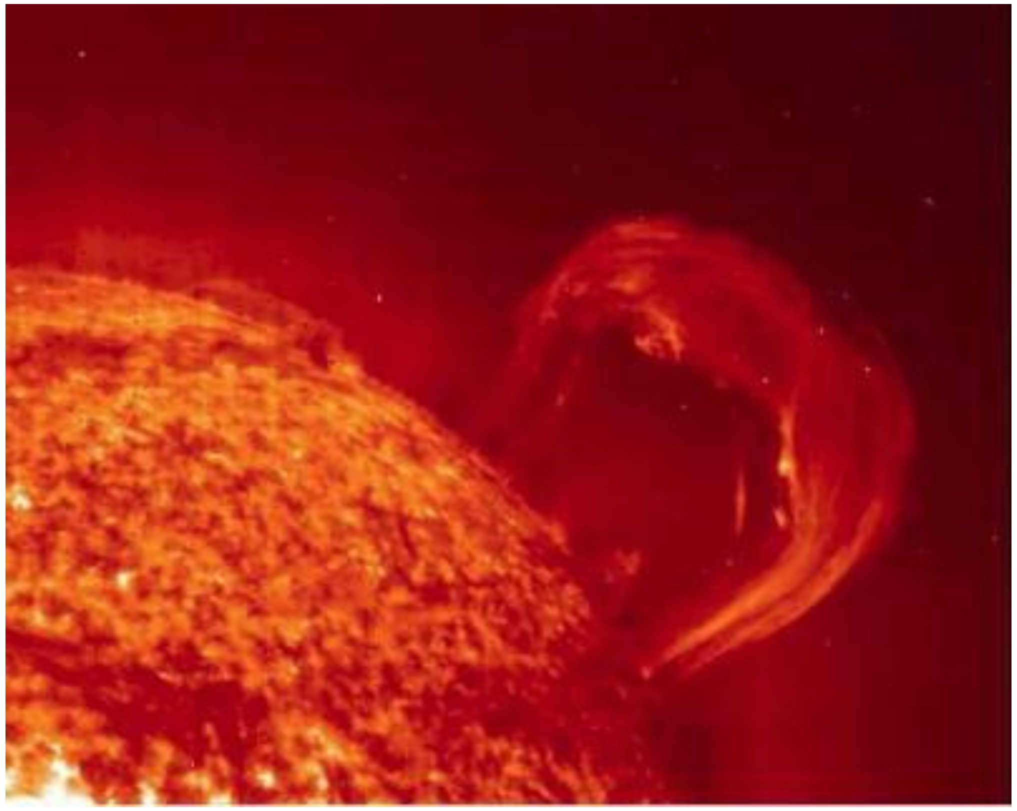
$$A = A_0(t\omega_{pe})^2$$

$$A_0 = \delta \mid \delta + \varepsilon_{\parallel} + \delta\Delta_{\kappa\parallel} \mid$$

In the magnetopause



On the solar surface



Conclusions & Future Work

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We found an analytic solution for arbitrary temperature and density gradients

- showing kinetic Biermann growth and temperature anisotropy growth solved for general T and n distributions

(Schoeffler et al. 2017 arXiv 1707.06069 and 1707.06390)

confirming the growth of the Biermann battery for collisionless systems (as a function of space)

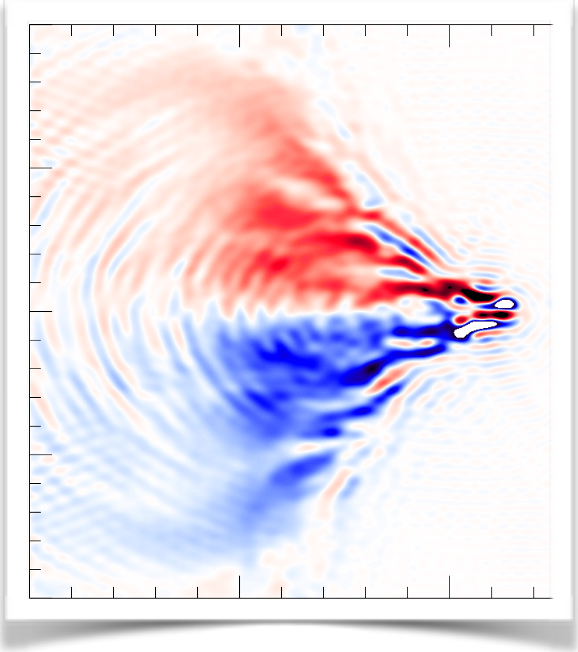
- linear growth proportional to $\nabla n \times \nabla T$

revealing anisotropy generation magnitude and direction (as a function of space)

- t^2 growth caused by ∇T (found for a general T and n distributions)

with results applicable to

- astrophysical magnetic field growth, and effects on heat flux
- laser experiments where collisions are weak



Relativistic laser simulations: N. Shukla

We found a kinetic solution!

Evolution of f

$$f = f_{\nabla T} + \frac{1}{2}\epsilon\delta\omega_{pe}t\frac{x}{\lambda_D}\bar{v_y}\left(5-\bar{v}^2\right)f_M + \frac{1}{2}\epsilon\delta\left(\omega_{pe}t\right)^2\bar{v_x}\bar{v_y}f_M$$

$$f_{\nabla T} \equiv f_0 + \frac{1}{2}\delta\omega_{pe}t\bar{v_y}\left(5-\bar{v}^2\right)f_M - \frac{1}{4}\delta^2\omega_{pe}t\frac{y}{\lambda_D}\bar{v_y}\left(25-12\bar{v}^2+\bar{v}^4\right)f_M + \delta^2\left(\omega_{pe}t\right)^2\left[\frac{1}{8}\bar{v_y}^2\left(39-14\bar{v}^2+\bar{v}^4\right)-\frac{1}{4}\left(5-\bar{v}^2\right)\right]f_M$$

E from Maxwell-Bolzmann potential

$$\frac{\mathbf{E}}{E_0} = -\left(\epsilon - \epsilon^2\frac{x}{\lambda_D} + \epsilon\delta\frac{y}{\lambda_D}\right)\hat{\mathbf{x}} - \delta\hat{\mathbf{y}}$$

Evolution of B

$$\frac{\mathbf{B}}{B_0} = -\epsilon\delta\omega_{pe}t\hat{\mathbf{z}}$$

(Schoeffler et al. 2017 arXiv 1707.06069)

$$\bar{\mathbf{v}} \equiv \frac{\mathbf{v}}{v_{T0}} \qquad E_0 \equiv m_e v_{T0}\omega_{pe}/e \qquad B_0 \equiv m_e c\omega_{pe}/e$$

What is a temperature anisotropy?

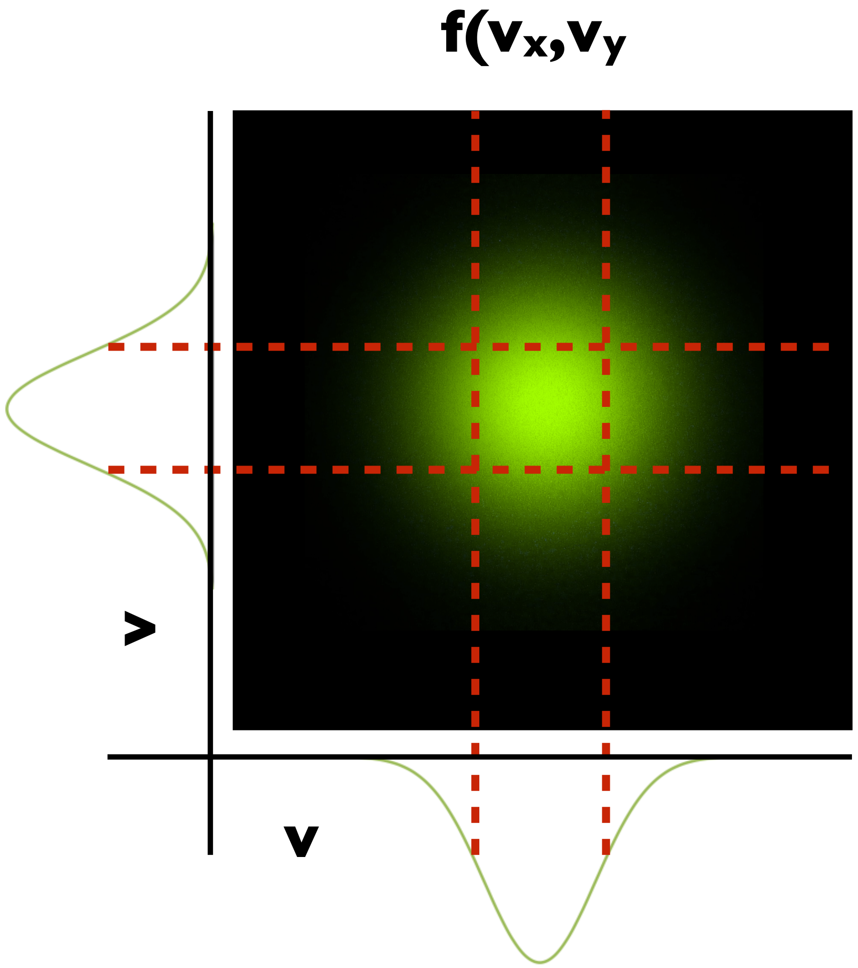
$$f = f\left(v_x, v_y, v_z, x, y, z, t\right)$$

$$T_{xx} = \frac{m_e}{n} \int dv^3 v_x^2 f$$

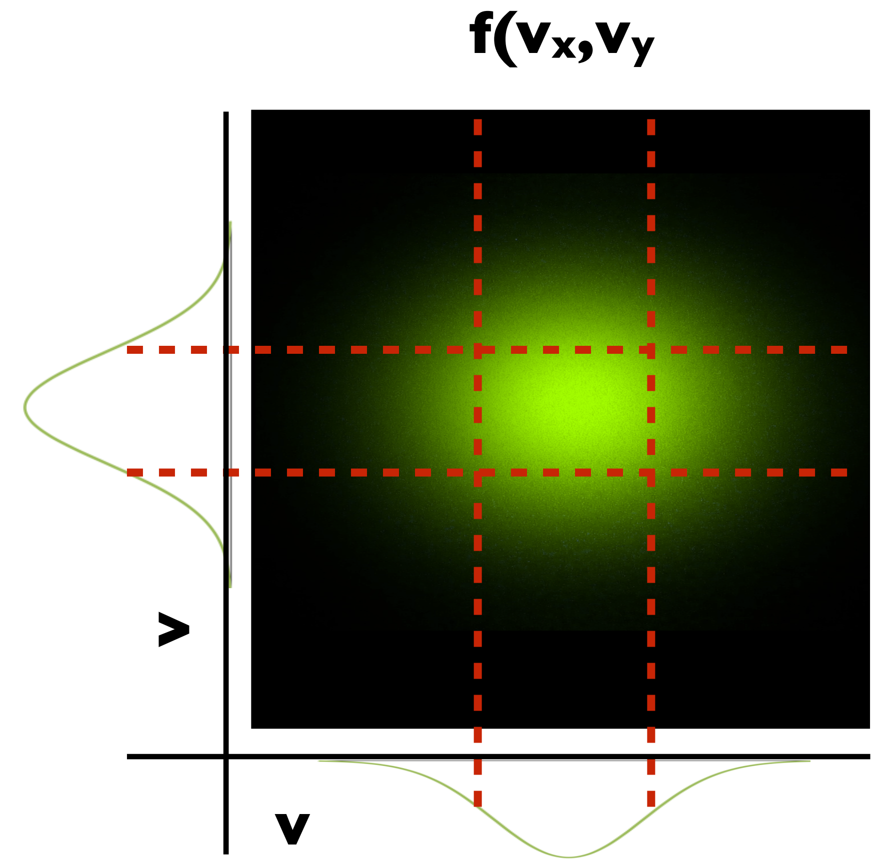
Maxwell distribution

$$f_M = n_0 \left(\frac{1}{2\pi v_{the}^2}\right)^{3/2} \exp\left(-\frac{1}{2}\frac{v^2}{v_{the}^2}\right)$$

Isotropic Temperature



Anisotropic Temperature



Most general temperature tensor in 2D

Rotated Temperature Tensor

$$T_{ij} = T_0 + T_0 \begin{vmatrix} 2(\delta^2(1+\kappa_\perp+\kappa_{||})+\varepsilon_{||}\delta)+A_0 & 0 \\ 0 & 2(\delta^2(1+\kappa_\perp+\kappa_{||})+\varepsilon_{||}\delta) - A_0 \end{vmatrix} (t\omega_{pe})^2/2$$

T_{ij} independent of κ_{nij}

Rotated Anisotropy

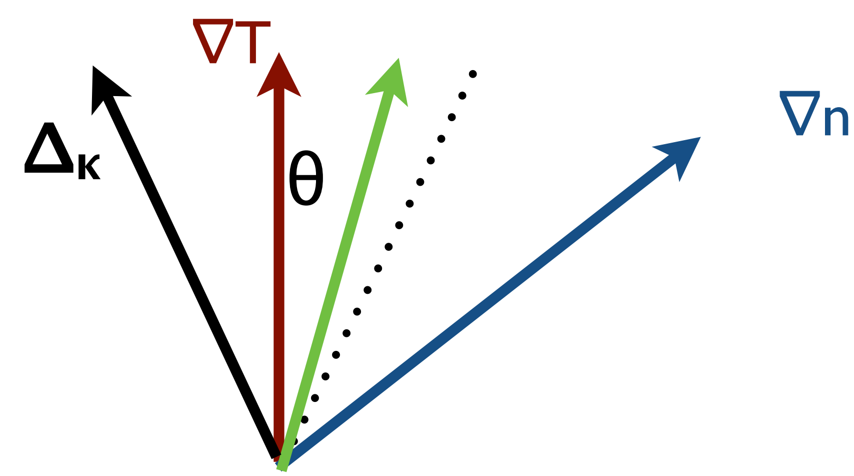
$$A = A_0(t\omega_{pe})^2$$

$$A_0 = \delta(\,\delta^2+\varepsilon^2+\delta^2\Delta_\kappa{}^2+2(\delta\varepsilon_{||}+\varepsilon_{||}\delta\Delta_{\kappa||}+\varepsilon_\perp\delta\Delta_{\kappa\perp}+\delta^2\Delta_{\kappa||}))^{1/2}$$

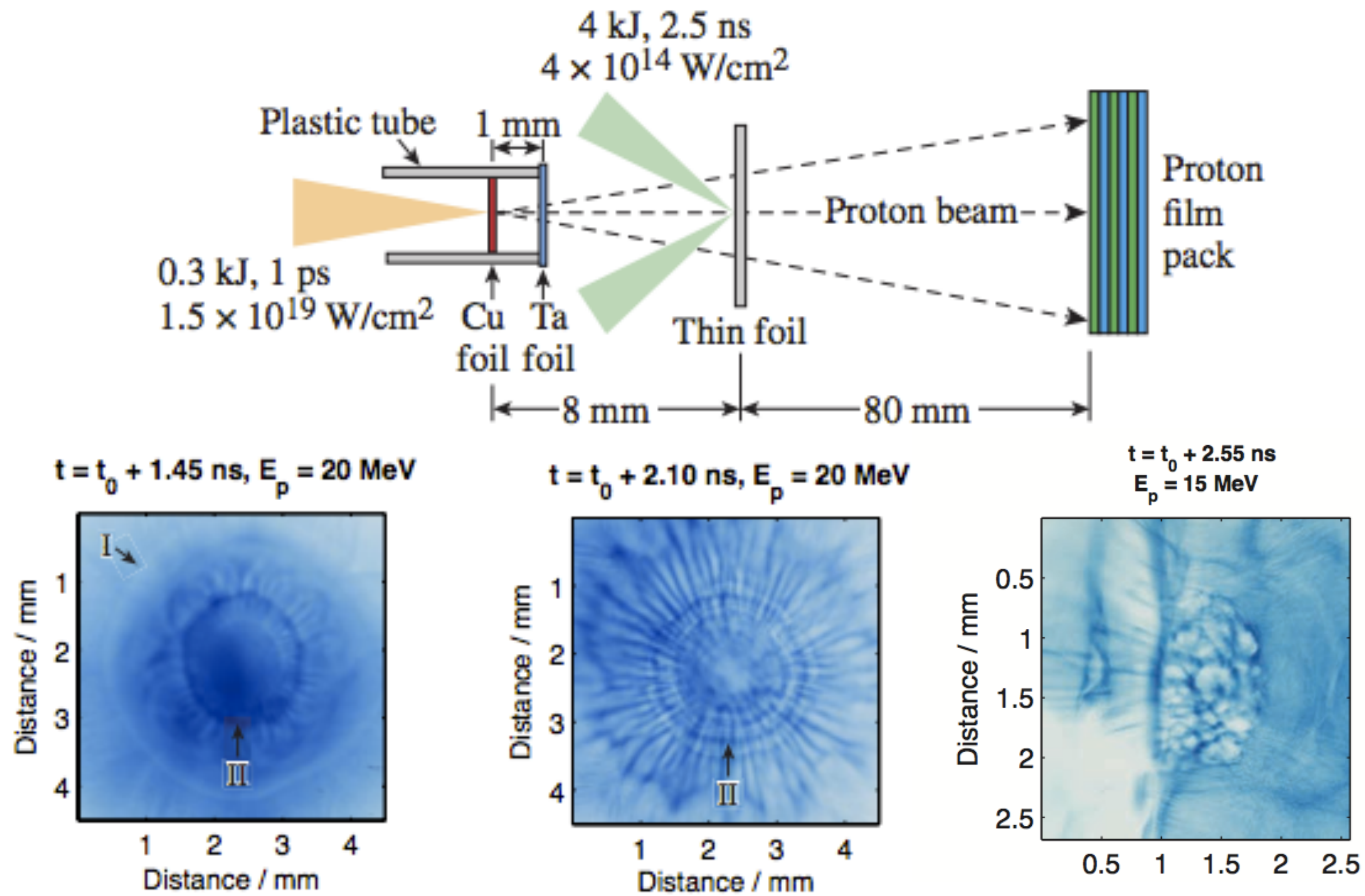
$$= \delta|\boldsymbol{\delta}+\boldsymbol{\varepsilon}+\delta\boldsymbol{\Delta}_\kappa|$$

rotated towards ∇n by

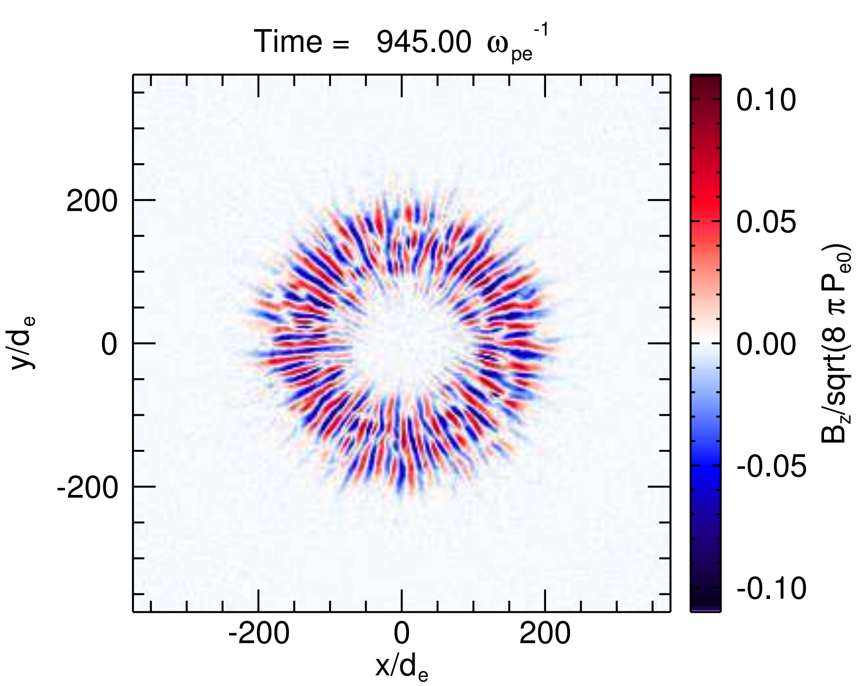
$$\theta = \frac{1}{2}\tan^{-1}\frac{\varepsilon_\perp+\delta\Delta_{\kappa\perp}}{\delta+\delta\Delta_{\kappa||}+\varepsilon_{||}}$$

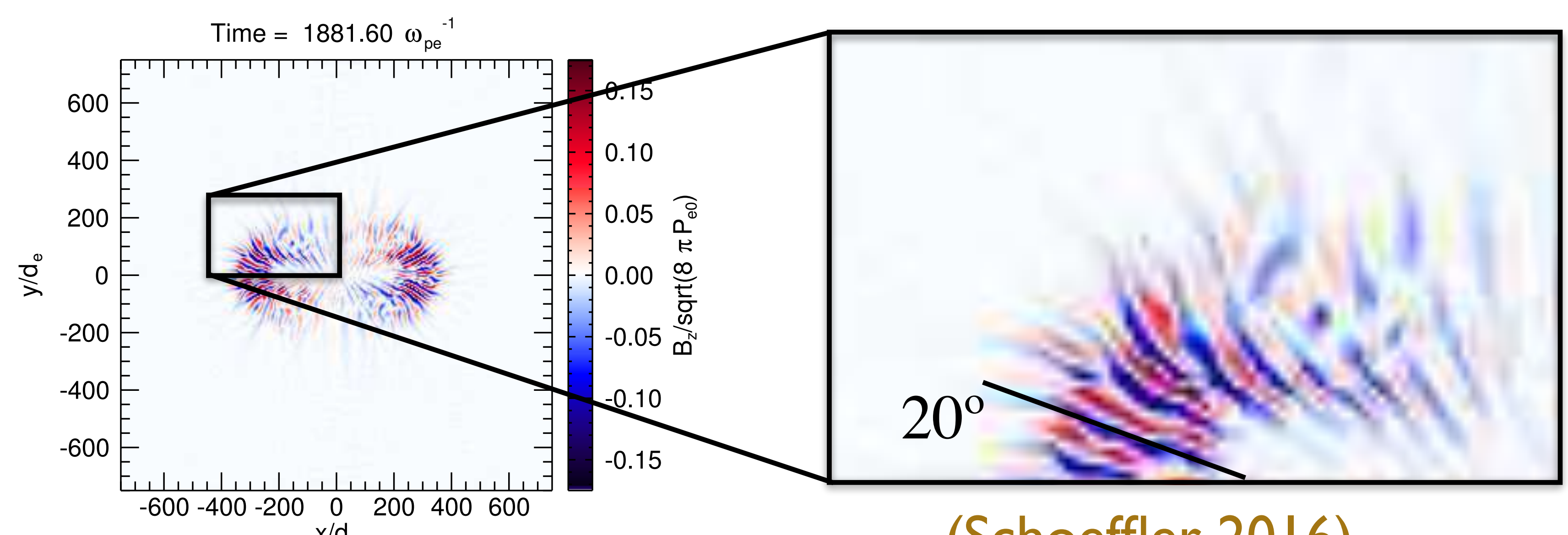


Maybe already seen in experiments



(Gao 2014)





(Schoeffler 2016)

Predicted kinetic Biermann field

$$\frac{eB}{m_e c \omega_{pe}} = \epsilon_\perp \delta \omega_{pe} t$$

Equal to fluid predictions

Equations also valid for anisotropic bi-Maxwellian distributions $T_\parallel \neq T_\perp$

$T_\parallel$ **parallel to ∇n**

Kinetic Biermann

$$B = -\frac{m_e c^3}{e} \frac{\nabla n \times \nabla T_\parallel}{m_e n c^2} t$$

Depends only on $\nabla T_\parallel$